# Some Example CFLs

$L_1 = \{a^n b^n \mid n \geq 0\}$

$G_1 = (\{S\}, \{a,b\}, R, S)$

$R: S \to aSb \mid \lambda$

$L_2 = \{a^n b^m \mid m \geq n\}$

$S \to aSb \mid Sb \mid \lambda$

$L_3 = \{a^n b^m \mid m > n\}$

$S \to aSb \mid Sb \mid b$

$L_4 = \{w w^R \mid w \in \{a,b\}^*\}$

$S \to aSa \mid bSb \mid \lambda$

NOTE $\{ww \mid w \in \{a,b\}^*\}$

IS <u>NOT</u> A CFL

$L_5 = \{w \mid \#a\text{'s in } w = \#b\text{'s in } w\}$

$S \to SaSbS \mid SbSaS \mid \lambda$

# A CHALLENGING CFL

THE COMPLEMENT OF $\{ww \mid w \in \{a,b\}^*\}$ IS A CFL. IT CAN BE DESCRIBED AS

$$\{xy \mid |x| = |y|, x \neq y\} \cup \{w \mid |w| \text{ IS ODD}\}$$

WHERE ALL STRINGS ARE OVER $\{a,b\}$

$\{w \mid w \in \{a,b\}^+ \text{ AND } |w| \text{ IS ODD}\}$

IS REGULAR

$$S \to aT \mid bT$$
$$T \to aS \mid bS \mid \lambda$$

WE WILL SHOW CFLS ARE CLOSED UNDER $\cup$

FOCUS ON

$$\{xy \mid |x| = |y|, x \neq y, x, y \in \{a,b\}^*\}$$

# CHALLENGE CONTINUED

$\{ xy \mid |x| = |y|\ x \neq y \}$

NOTE THAT WHILE WW REQUIRES THAT THERE BE <u>NO</u> ERRORS TRANSCRIBING (COPYING) FIRST HALF TO SECOND HALF, ITS COMPLEMENT REQUIRES JUST <u>ONE</u> TRANSCRIPTION ERROR. ELEMENTS ARE OF FORM

$$x_1 a x_2 y_1 b y_2 \text{ OR } x_1 b x_2 y_1 a y_2$$

WHERE $|x_1| = |y_1|$ AND $|x_2| = |y_2|$

NOTE THAT WE DO NOT CARE WHAT $x_1, x_2, y_1$ AND $y_2$ ARE. ONLY THEIR LENGTHS MATTER. WITH THAT INSIGHT IN MIND, WE CAN SEE THAT

$$x_1 a y_1 x_2 b y_2 \text{ OR } x_1 b y_1 x_2 a y_2$$

WHERE $|x_1| = |y_1|$ AND $|x_2| = |y_2|$ ALSO DESCRIBES ELEMENTS IN LANGUAGE. THIS IS JUST

$$\left.\begin{array}{l} S \to AB \mid BA \\ A \to CAC \mid a \\ B \to CBC \mid b \end{array}\right\} \quad (\{S,A,B,C\},\{a,b\},R,S)$$

$$C \to a \mid b$$

WHICH IS A CFG.

# SIMPLE CLOSURES

LET $G_1 = (V_1, \Sigma, R_1, S_1)$ AND $G_2 = (V_2, \Sigma, R_2, S_2)$

WHERE $V_1 \cap V_2 = \emptyset$

LET $L_1 = \mathcal{L}(G_1)$ AND $L_2 = \mathcal{L}(G_2)$

WHERE $G_1$ AND $G_2$ ARE CFGs AND SO

$L_1$ AND $L_2$ ARE CFLs

UNION: $G_3 = (V_1 \cup V_2 \cup \{S\}, \Sigma, R_3, S)$

$R_3$: $S \to S_1 \mid S_2$

ADD $R_1$ AND $R_2$ TO $R_3$ AND GET

$L_1 \cup L_2$

CONCATENATION:

$R_3$: $S \to S_1 S_2$

ADD $R_1$ AND $R_2$ TO $R_3$ AND GET

$L_1 \circ L_2$

STAR:

$R_3$: $S \to S S_1 \mid \lambda$

ADD $R_1$ TO $R_3$ AND GET

$L_1^*$

# NON-CLOSURES

ASSUME, FOR NOW (PUMPING LEMMA WILL VERIFY) THAT

$$L = \{a^n b^n c^n \mid n \geq 0\}$$

IS NOT A CFL

NOW, CONSIDER TWO CFLs

$$L_1 = \{a^n b^n c^m \mid n, m \geq 0\}$$
$$= a^n b^n c^*$$

AND $L_2 = \{a^m b^n c^n \mid n, m \geq 0\}$

$$= a^* b^n c^n$$

$$L_1 \cap L_2 = L = \{a^n b^n c^n \mid n \geq 0\}$$

BUT $L_1 = \mathcal{L}(G_1)$, $L_2 = \mathcal{L}(G_2)$ WHERE

$G_1 = (\{S,T\}, \{a,b,c\}, R_1, S)$ $G_2 = (\{S,T\}, \{a,b,c\}, R_2, S)$

$R_1$: $S \rightarrow Sc \mid T$
$T \rightarrow aTb \mid \lambda$

$R_2$: $S \rightarrow aS \mid T$
$T \rightarrow bTc \mid \lambda$

SO CFLs NOT CLOSED UNDER $\cap$

PREVIOUSLY WE SHOWED NOT UNDER COMPLEMENT, IF WE CAN SHOW

$$L = \{ww \mid w \in \{a,b\}^*\}$$

IS NOT A CFL (LATER)