Graph Theory Problems

Undirected Acyclic Graphs

1. Coloring Problem

2. Independent Set Problem

3. Vertex Covering Problem
   (Really about Edges)

Vertex Covering

Choose set of vertices such that every edge has an associated selected vertex

Optimization Version

What is minimum size set?

Decision Problem

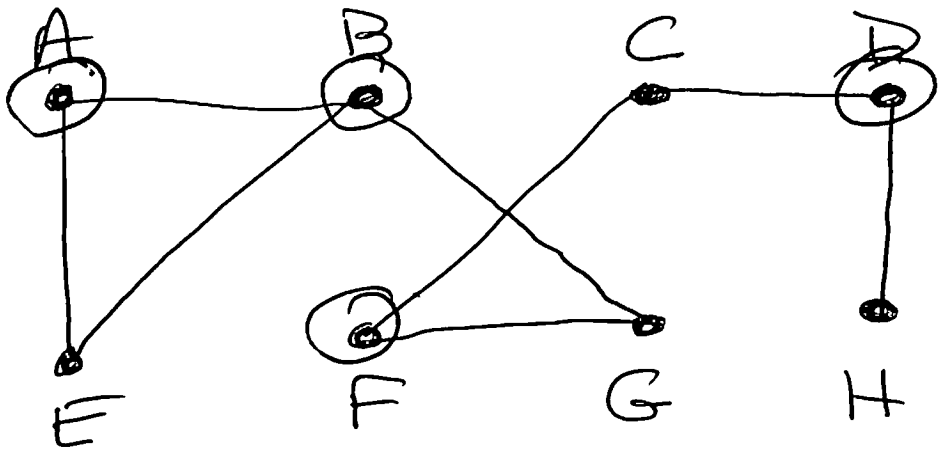
Given $k > 0$, can the graph be covered by k vertices

Can solve Optimization in Poly Time if can solve Decision in Poly

Try $1, 2, \ldots, n$ $n = \#$ Vertices

Note: n always works

VC K=4



A, B, F, D COVERS

OR

E, B, F, D COVERS

Independent Set

Choose set of vertices such that no two vertices have common edge (are connected)

Optimization Version

What is Maximum Size Set?

Decision Problem

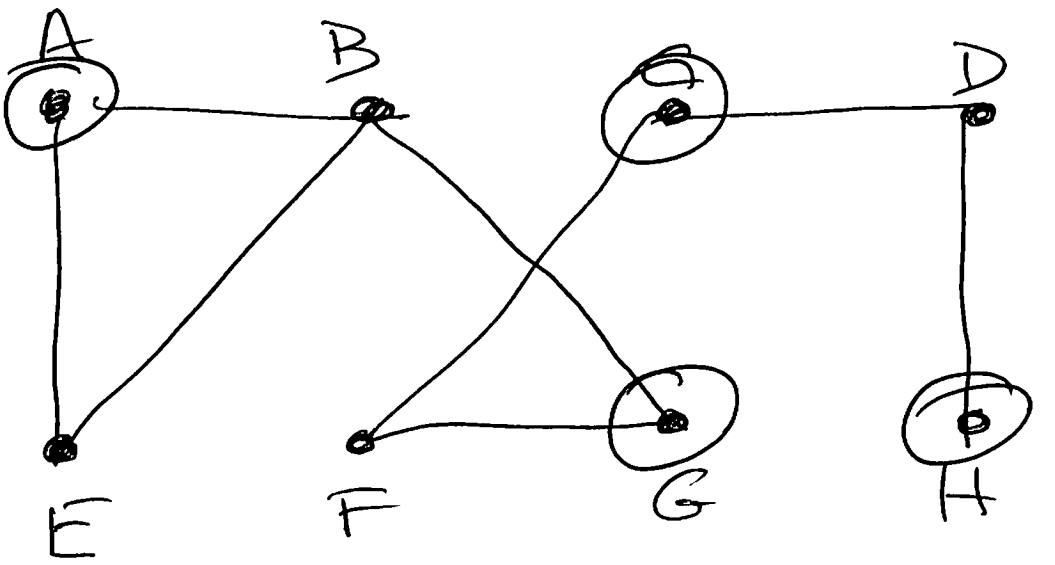
Given $k > 0$, can the graph have an independent set of size k

Can solve optimization in poly time if can solve decision in poly

Try $n, n-1, \ldots, 1$ $n = \#$ vertices

Note! 1 always works

IS $k=4$



A G C H IS IS

OR

E G C H IS IS

Graph Coloring

Color each vertex so no neighbor vertex has same color

Optimization

What is minimum # of colors

Decision

Given $k>0$, can we color with k colors

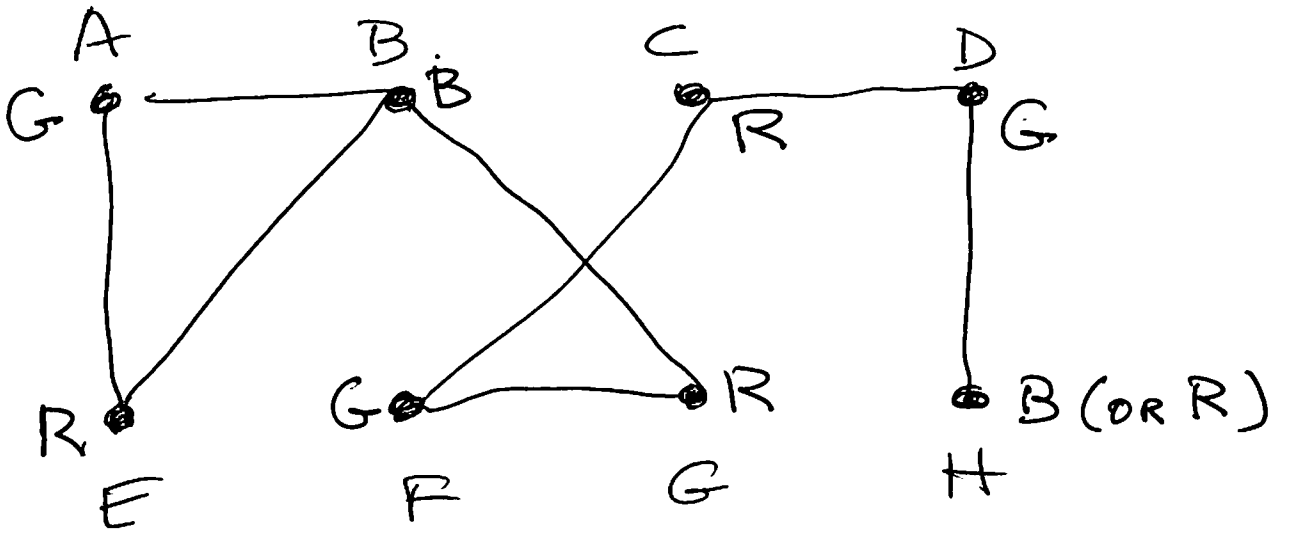
Can solve optimization in poly if decision solvable in poly

Try $1, 2, \ldots, n = \#$ vertices

n always works

4 always works if planar (no crossing edges)

GC $k=3$



ABOVE IS ONE OF MANY 3-COLOR SOLUTIONS