CKY: $G = (\{S, A, B, C, D, E, F, G, H\}, \{a, b\}, R, S)$

R: $S \to AB \mid AC \mid AD \mid AE$
$\mid BA \mid BF \mid BG \mid BH$

$A \to a$ $\quad C \to DS \quad F \to GS$
$B \to b$ $\quad D \to SB \quad G \to SA$
$E \to BS \quad H \to AS$

DOES babba BELONG TO $\mathcal{L}(G)$?

	b	a	b	b	a
1	B	A	B	B	A
2	S	S	$\emptyset$	S	
3	E, D	D	E		
4	$\emptyset$	S			
5	E, C				

$\leftarrow$ $babba \notin \mathcal{L}(G)$

COCKE-KASAMI-YOUNGER
CKY

$S \to AB \mid AC \mid AD \mid AE$
$\quad \mid BA \mid BF \mid BG \mid BH$

$A \to a$ $\quad C \to DS$ $\quad F \to GS$
$B \to b$ $\quad D \to SB$ $\quad G \to SA$
$\quad E \to BS$ $\quad H \to AS$

	b	a	b	b	a	a	b	a
1	B	A	B	B	A	A	B	A
2	S	S		S		S	S	
3	E,D	D	E	G	H	H,G		
4		S	S	S				
5	E,C	H,G	E,D	F,G				
6	S	S	S					
7	E,C,D	H,F,G						
8	S							

ACCEPT

PUMPING LEMMA FOR CFLs

IF L IS A CFL, THEN $\exists N > 0$, SUCH THAT IF $z \in L$ AND $|z| \geq N$ THEN $z = uvwxy$, WHERE $|vwx| \leq N$, $|vx| > 0$ AND $\forall i \geq 0$ $uv^iwx^iy \in L$

NOTE THAT IF $|\Sigma| = 1$ THEN PUMPING LEMMA FOR CFL DEGENERATES TO PUMPING LEMMA FOR REGULAR

THUS, IF $L \subseteq \Sigma^*$, $|\Sigma| = 1$, AND L IS NOT REGULAR THEN L IS NOT A CFL

$\{ww \mid w \in \{a,b\}^*\}$ IS NOT A CFL

1. ASSUME L IS A CFL
2. PL GIVES YOU N
3. YOU CHOOSE $a^N b^N a^N b^N$
4. PL SPLITS $a^N b^N a^N b^N$ INTO $uvwxy$
   WHERE $|vwx| \leq N$, $|vx| > 0$
   AND STATES $\forall i \geq 0$ $uv^iwx^iy \in L$
5. a.) IF vwx IS OVER a'S ONLY
   THEN SINCE $|vwx| \leq N$
   vwx CANNOT SPAN FROM ONE OF
   THE a^N SUBSTRINGS TO OTHER,
   AND SO FOR $i=0$, WE GET EITHER
   $a^{N-|vx|} b^N a^N b^N \in L$ OR $a^N b^N a^{N-|vx|} b^N \in L$
   AND BOTH ARE NOT SO.

   b) IF vwx IS OVER b'S ONLY, THE
   SAME ARGUMENTS APPLY

   c) IF vwx SPANS a'S AND b'S THEN
   THIS MUST BE WITHIN ONE OF
   THE $a^N b^N$ OR $b^N a^N$ SUBSTRINGS
   AS $|vwx| \leq N$. IF $i=0$ THEN WE
   REDUCE SOME a'S AND SOME b'S, BUT NOT ALL,
   WITHIN JUST ONE OF THE CONSECUTIVE
   a'S AND b'S, BUT NOT THE OTHER
   AND SO THE RESULTING STRING IS NOT
   IN L.

$\times$

PUSHDOWN AUTOMATA
PDA

$$A = (Q, \Sigma, \Gamma, \delta, q_0, Z_0, F)$$

STATES (FINITE)

LANG. ALPHABET (FINITE)

STACK SYMBOLS (FINITE

START STATE

OPTIONAL BOTTOM OF STACK MARKER

OPTIONAL FINAL STATES

TRANSITION FUNCTION

$$\delta : Q \times \Sigma_e \times \Gamma_e \rightarrow 2^{Q \times \Gamma^*}$$

CAN EXTEND TO Γ^* CAN LIMIT TO Γ_e BY GROWING STATES

PDA DIAGRAMS

SOME TEXTS USE $a, X / Y$ WHICH IS FINE ALSO

$$\textcircled{S} \xrightarrow{a, \alpha \to \beta} \textcircled{P}$$

$$\delta(s, a, \alpha) \supseteq \{(p, \beta)\}$$

PDA LANGUAGES
$\equiv$ CFLs

INSTANTANEOUS DESCRIPTIONS (ID)

$[q, w, \gamma]$

q - CURRENT STATE
w - REMAINING INPUT
γ - STACK CONTENTS
READ LEFT (TOP) TO RIGHT (BOTTOM)

SINGLE STEP

$[q, ax, Z\alpha] \vdash [p, x, \beta\alpha]$ IF $\delta(q, a, Z) \in (p, \beta)$

MULTISTEP $\vdash^*$ REFLEXIVE TRANSITIVE CLOSURE OF $\vdash$

GIVEN $A = (Q, \Sigma, \Gamma, \delta, q_0, Z_0, F)$
THERE CAN BE THREE NOTIONS OF ACCEPTANCE

FINAL STATE

$\mathcal{L}(A) = \{w \mid [q_0, w, Z_0] \vdash^* [f, \lambda, \beta]\}, f \in F$

EMPTY STACK

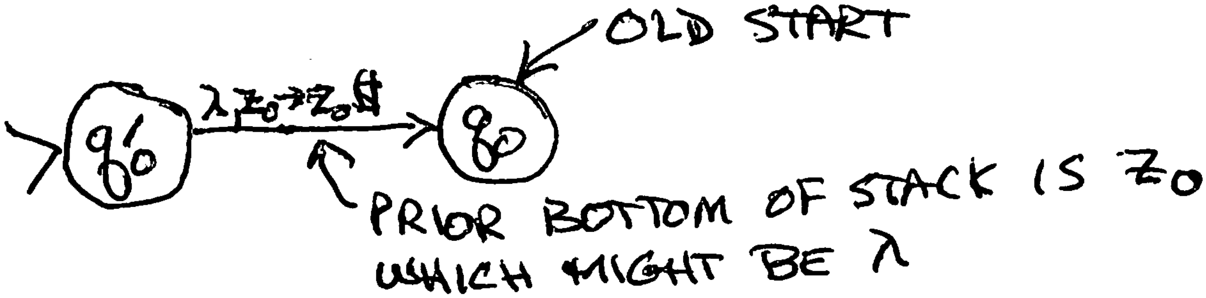
$N(A) = \{w \mid [q_0, w, Z_0] \vdash^* [q, \lambda, \lambda]\}, q \in Q$

EMPTY STACK AND FINAL STATE

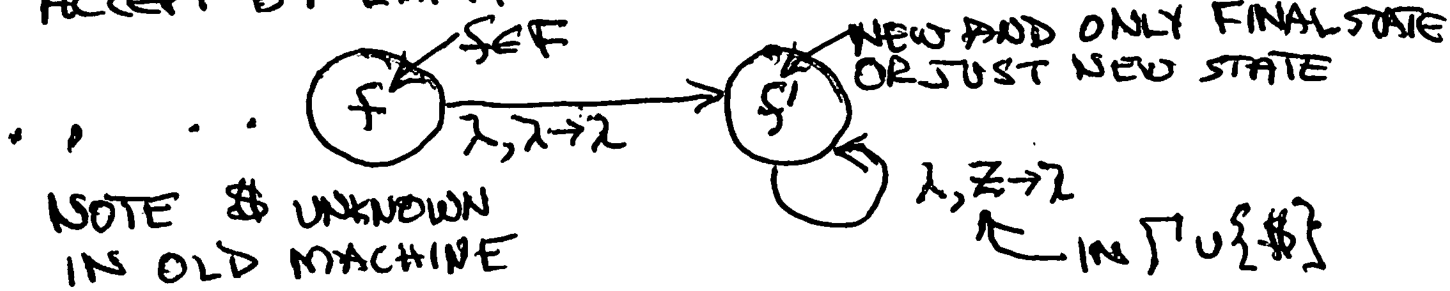
$E(A) = \{w \mid [q_0, w, Z_0] \vdash^* [f, \lambda, \lambda]\}, f \in F$

EQUIVALENCY OF LANGUAGE CLASSES, $\mathcal{L}(A)$, $N(A)$, $E(A)$, WHERE A RANGES OVER ALL PDAs

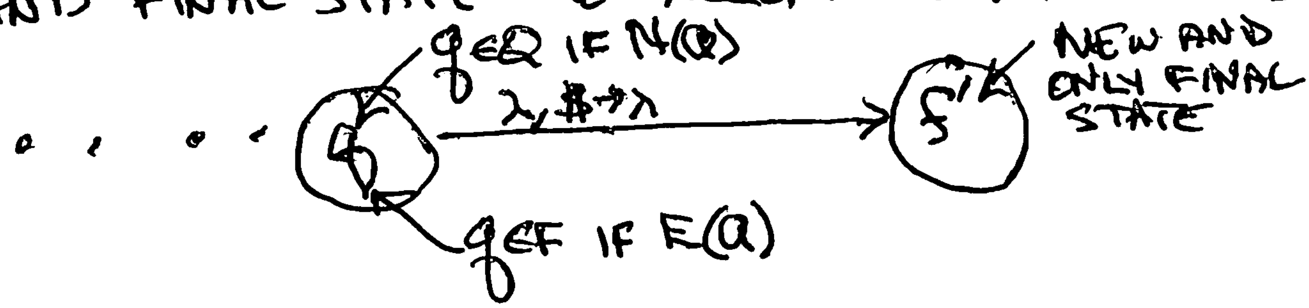
- CONVERTING ONE FORM TO ANOTHER
  FOR EACH CASE, ASSUME q_0' IS A NEW STATE (NEW START) AND $ IS A NEW STACK SYMBOL PLACED ON BOTTOM OF STACK. FOR ALL CASES, START WITH



- CHANGE ACCEPT BY FINAL STATE TO ACCEPT BY EMPTY STACK OR EMPTY AND FINAL STATE



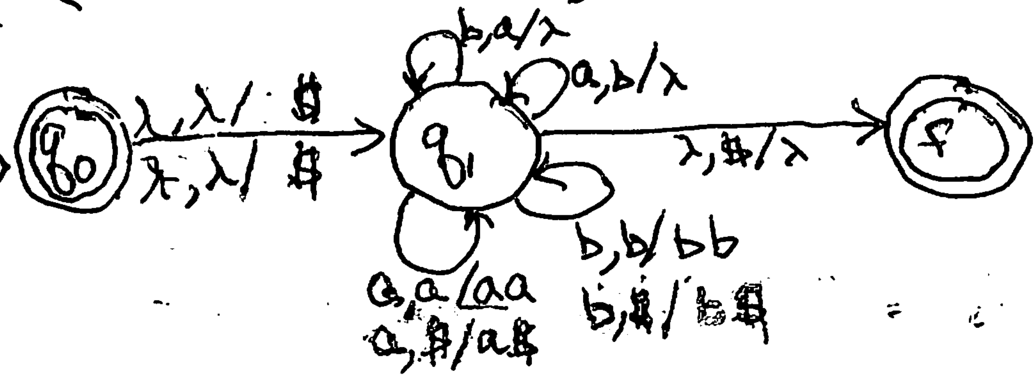
- CHANGE ACCEPT BY EMPTY STACK OR EMPTY AND FINAL STATE TO ACCEPT BY FINAL STATE



EXAMPLE PDA

$L_1 = \{ w \mid |w|_a = |w|_b \}$. ASSUME λ ON STACK

Q_1:



I USED / WHERE BOOK USES $\rightarrow$

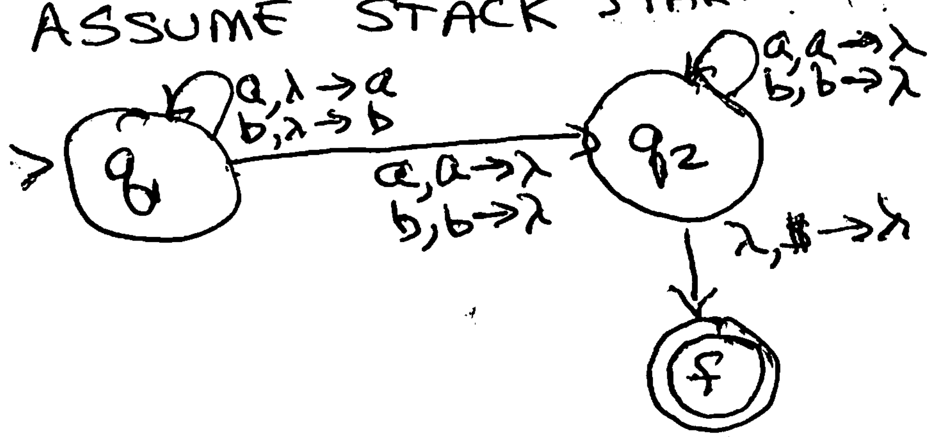
THIS GETS $L_1 = E(Q_1)$

THIS IS NON-DETERMINISTIC

$L_2 = \{ ww^R \mid w \in \{a, b\}^+ \}$

ASSUME STACK STARTS WITH \$

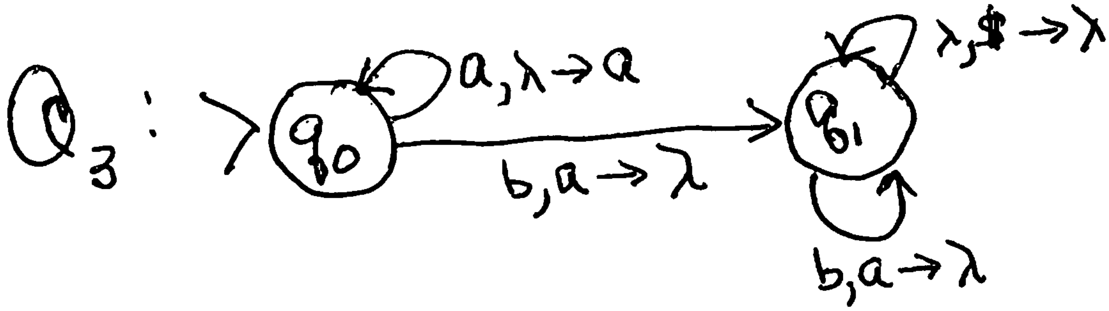
Q_2:



THIS GETS $L_2 = \mathcal{L}(Q_2) = E(Q_2)$

THIS IS ALSO NON-DETERMINISTIC

$L_3 = \{a^n b^n \mid n > 0\}$. ASSUME \$ ON STACK



THIS GETS $L_3 = N(Q_3)$

THIS IS DETERMINSTIC

DETERMINISM FOR PDAs

1) FOR EACH $q \in Q$ & $z \in \Gamma$ & $a \in \Sigma$
IF $|\delta(q, \lambda, z)| > 0$ THEN $|\delta(q, a, z)| = 0$

2) FOR NO $q \in Q$, $z \in \Gamma$ & $a \in \Sigma_\epsilon$
IS $|\delta(q, a, z)| > 1$

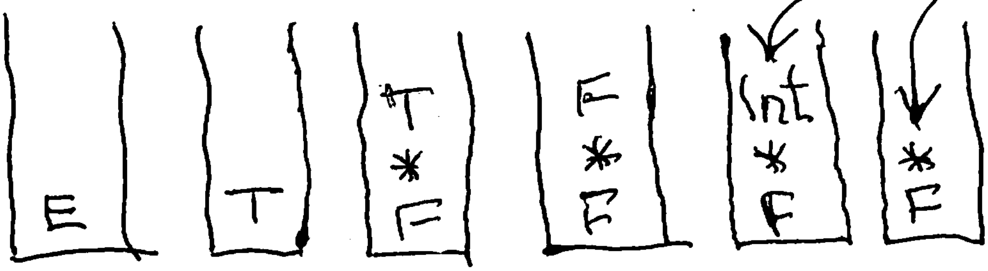
TOP-DOWN PARSER

$E \to E+T \mid T$

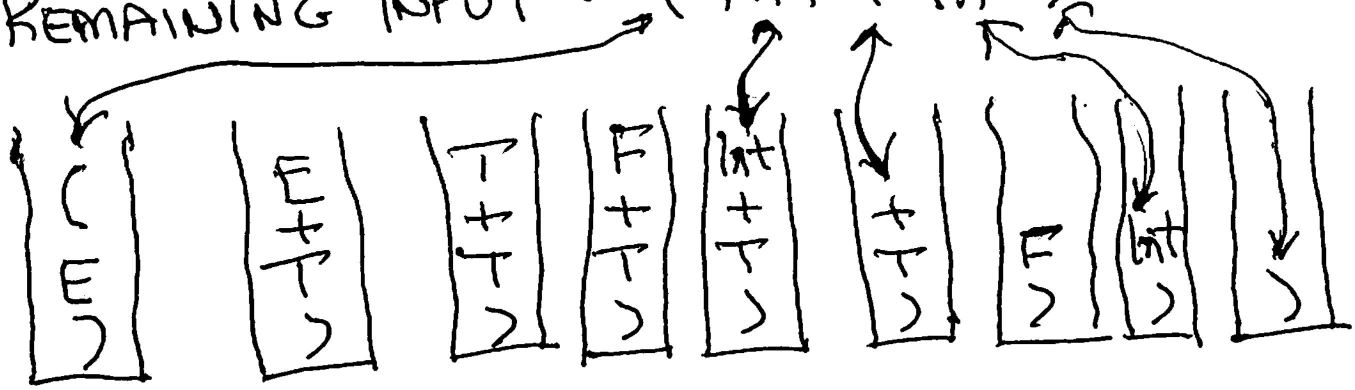
$T \to T*F \mid F$

$F \to (E) \mid \text{int}$

INPUT: $7 * (3+21) \Rightarrow \text{int} * (\text{int}+\text{int})$



REMAINING INPUT: (Int + Int)

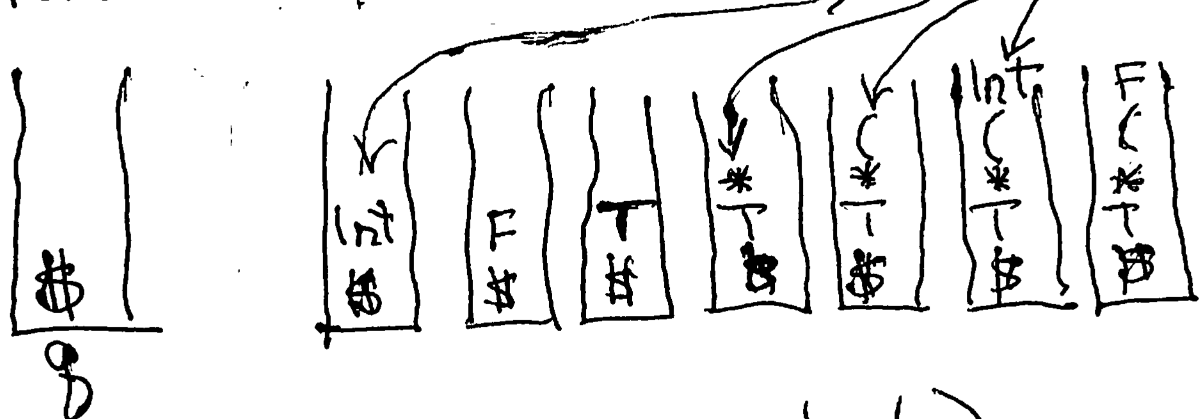


ACCEPTS BY EMPTY STACK

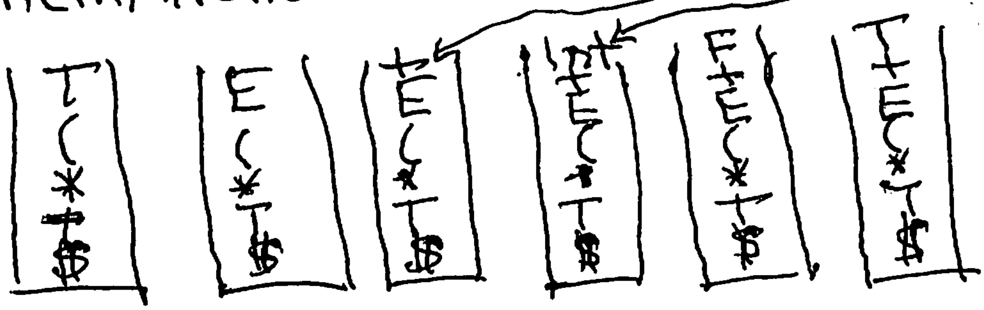
BOTTOM-UP PARSER

$E \rightarrow E + T \mid T$
$T \rightarrow T * F \mid F$
$F \rightarrow (E) \mid Int$

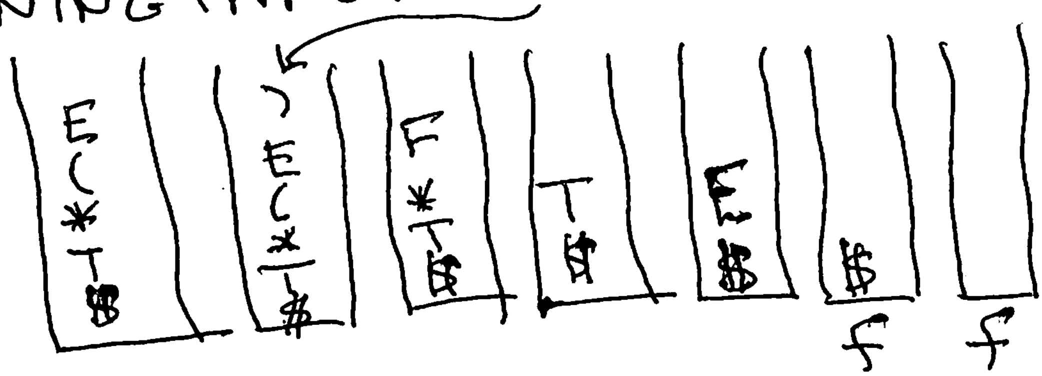
INPUT: $7 * (3+21) \Rightarrow Int * (Int + Int)$



REMAINING INPUT $+ Int)$



REMAINING INPUT $)$



ACCEPTS BY FINAL STATE AND
EMPTY STATE
OR JUST EMPTY STACK (CAN ALTER FOR FINAL)

WEEK 9

CFLs

CLOSURE

EASY:

CONCATENATION: $L_A \cdot L_B$

G'_{CAT} $S \to S_A S_B$

S_A: START FOR L_A

S_B: START FOR L_B

KLEENE STAR: L_A^*

G'_{STAR} $S \to S_A S \mid \lambda$

S_A: START FOR L_A

NOTE: $G_A = (V_A, \Sigma, R_A, S_A)$

$G_B = (V_B, \Sigma, R_B, S_B)$

$G_{CAT} = (\{S\} \cup V_A \cup V_B, \Sigma, \{S \to S_A S_B\} \cup R_A \cup R_B, S)$

$G_{STAR} = (\{S\} \cup V_A, \Sigma, \{S \to S_A S \mid \lambda\} \cup R_A, S)$

CFL CLOSURE

EASY: UNION

$G_A = (V_A, \Sigma, R_A, S_A)$

$G_B = (V_B, \Sigma, R_B, S_B)$

$G = (\{S\} \cup V_A \cup V_B, \Sigma, \{S \to S_A | S_B\} \cup R_A \cup R_B, S]$

MODERATE: INTERSECTION WITH REGULAR

$Q_0 = (Q_0, \Sigma, \Gamma, \delta_0, q_0, \$, F_0)$ PDA, $L_0 = L(Q_0$

$Q_1 = (Q_1, \Sigma, \delta_1, q_1, F_1)$ DFA $= L_1 = L(Q$

DEFINE

$Q_2 = (Q_0 \times Q_1, \Sigma, \Gamma, \delta_2, \langle q_0, q_1 \rangle, \$, F_0 \times F_1)$

$\delta_2(\langle q, s \rangle, a, X) \supseteq \{(\langle q', s' \rangle, \alpha)\}, X \in \Gamma, a \in \Sigma_\epsilon$

IFF $\delta_0(q, a, X) \supseteq \{(q', \alpha)\}$

AND $\delta_1(s, a) = s'$ IF $a \in \Sigma$

$\delta_1(s, a) = s$ IF $a = \lambda$ TREAT AS NO STATE $s' = s$

ELSE $\delta_1(s, a) = s$

NOW INDUCTIVELY CAN SHOW

$[\langle q_0, q_1 \rangle, w, \$] \vdash^* [\langle t, s \rangle, \lambda, B]$ IFF

$[q_0, w, \$] \vdash^* [t, \lambda, B]$ IN Q_0

AND $[q_1, w] \vdash^* [s, \lambda]$ IN Q_1

SO $w \in F(Q_2)$ IFF $t \in F_0$ & $s \in F_1$ IFF $w \in F(Q_0) \cap F(Q_1)$

WEEK 9

CFL CLOSURE

MODERATE: SUBSTITUTION

$G = (V, \Sigma, R, S) \quad L = \mathcal{L}(G)$

SUBSTITUTION

$f(a) = L_a \quad a \in \Sigma \quad L_a$ A CFL

$G_a = (V_a, \Sigma, R_a, S_a) \quad L_a = \mathcal{L}(G_a)$

ASSUME SAME OR USE Σ_a

IN R, CHANGE ALL INSTANCES OF $a \in \Sigma$ IN RHS's TO S_a

THUS, IF ORIGINALLY,

$S \Rightarrow a_1, \ldots, a_k$

THEN, IN NEW

$S \overset{*}{\Rightarrow} S_{a_1} \cdots S_{a_k}$

AND THEN

$S \overset{*}{\Rightarrow} w_{a_1} \cdots w_{a_k}$

WHERE $w_{a_i} \in f(a_i)$

MAY NEED $\bigcup_{a \in \Sigma} \Sigma_a$

$G' = (V \cup V_A \cup V_B, \Sigma, R', S)$

$R' = R^{\text{CHANGED}} \cup R_a \quad a \in \Sigma$

WHERE R^{CHANGED} IS AS ABOVE

WEEK 9
CFL NON-CLOSURE

INTERSECTION

BY EXAMPLE OF CONTRADICTORY CASE

$L_1 = \{a^n b^n c^m \mid n,m > 0\}$

$L_2 = \{a^m b^n c^n \mid n,m > 0\}$

$S_1 \to S_1 c \mid T_1 c$ $\quad$ $S_2 \to a S_2 \mid a T_2$

$T_1 \to a T_1 b \mid a b$ $\quad$ $T_2 \to b T_2 c \mid b c$

BOTH ARE CFLs

HOWEVER,

$L_1 \cap L_2 = \{a^n b^n c^n \mid n > 0\}$

WHICH IS NOT A CFL

COMPLEMENT

BY FACT THAT CLOSURE UNDER UNION AN COMPLEMENT IMPLIES CLOSURE UNDER INTERSECTION

$\sim(\sim A \cup \sim B) = A \cap B$

COMPLEMENT EXAMPLE FOR CFL

$L = \{ww \mid w \in \{a,b\}^+\}$
IS NOT A CFL BUT

$\overline{L} = \{Z \mid |Z| \text{ IS ODD AND } Z \in \{a,b\}^+\}$
$\cup \{XY \mid |X| = |Y| \text{ AND } X \neq Y\}$

FIRST PART IS REGULAR
SECOND PART IS SEEN AS

$X_1 a X_2 Y_1 b Y_2$ OR $X_1 b X_2 Y_1 a Y_2$
WHERE $|X_1| = |Y_1|$, $|X_2| = |Y_2|$
BUT X_1, X_2, Y_1, Y_2 VALUES ARE UNIMPORTANT
SO LOOK AT AS

$X_1 a \underbrace{Y_1 X_2}_{\text{LENGTH ONLY}} b Y_2$ AND $X_1 b \underbrace{Y_1 X_2}_{\text{LENGTH ONLY}} b Y_2$

$S \to AB \mid BA$
$A \to XAX \mid a$
$B \to XBX \mid b$
$X \to a \mid b$

LOTS OF CLOSURE

SUBSTITUTION & INTERSECTION W/REGULAR

$f(a) = \{a, a'\}$

PREFIX

$g(a) = \{a'\}$

$h(a) = \{a\}; \; h(a') = \{\lambda\}$

INFIX

$L/R = h(f(L) \cap (\Sigma^* \cdot g(R)))$

SUFFIX

$x \cdot y' \quad x \in \Sigma^*, \; y \in R$

QUOTIENT WITH REGULAR

$x \cdot y' \quad xy \in L, \; y \in R$

ETC.

BUT <u>NOT</u>

QUOTIENT WITH CFL

WEEK 9

MIN AND MAX

$MAX(L_1) = \{x \mid x \in L \text{ AND } xy \in L \text{ IF } |y| > 0\}$

$MIN(L_2) = \{x \mid x \in L \text{ AND NO PROPER PREFIX OF } x \in L\}$

$L_1 = \{a^i b^j c^k \mid k \leq i \text{ OR } k \leq j\}$

$MAX(L_1) = \{a^i b^j c^k \mid k = MAX(i,j)\}$ NOT CFL

$MIN(L_1) = \{\lambda\}$ REGULAR

$L_2 = \{a^i b^j c^k \mid k > i \text{ OR } k > j\}$

$MAX(L_2) = \{\}$ REGULAR

$MIN(L_2) = \{a^i b^j c^k \mid k = MIN(i,j)+1\}$ NON-CFL

BOTH L_1 AND L_2 ARE CFLs

NON-CLOSURE HERE

MIN + MAX OF CFLs

$L_1 = \{a^i b^j c^k \mid k \le i \text{ OR } k \le j\}$

$L_2 = \{a^i b^j c^k \mid k \ge i \text{ OR } k \ge j\}$

$MAX(L_1) = \{a^i b^j c^k \mid k = MAX(i,j)\}$

$MIN(L_1) = \{\lambda\}$

$MAX(L_2) = \{\ \} = \emptyset$

$MIN(L_2) = \{a^i b^j c^k \mid k = MIN(i,j)\}$

$MAX(L_1)$ IS <u>NOT</u> A CFL

$MIN(L_2)$ IS <u>NOT</u> A CFL

SHOWS CFLs ARE <u>NOT</u> CLOSED UNDER EITHER MAX OR MIN

NOTE: REGULAR ARE CLOSED UNDER MAX AND MIN

CHALLENGING LANGUAGES

$L_1 = \{a^i b^j c^k \mid k = \min(i,j)\}$
$L_2 = \{a^i b^j c^k \mid k = \max(i,j)\}$

CONSIDER L_1 AND LET $N > 0$ BE FROM PL
WE MIGHT CHOOSE $a^N b^N c^N$
PL SPLITS IN $uvwxy \ni |vwx| \leq N, |vx| > 0$
AND SAYS $\forall i \; uv^iwx^iy \in L$

WE CHOOSE $i = 0$
CASE 1 IF vx CONTAINS AT LEAST ONE 'a',
WE SEE THAT $uwy \notin L$ AS IT HAS
FEWER 'a''s THAN 'c''s SINCE CONSTRAINT
$|vwx| \leq N$ MEANS vx CANNOT CONTAIN
ANY 'c''s IF IT CONTAINS 'a''s

CASE 2 ASSUME vx CONTAINS NO 'a''s THEN IT
CONTAINS AT LEAST ONE 'b' OR AT LEAST
ONE 'c'. NASTY CASE: IT CONTAINS
THE SAME NUMBER OF b's AS c's
IN WHICH CASE $uwy \in L$

FAIL!!!

WE CONSTRAINED OURSELVES

STATEMENT ABOUT z, $|z| \geq N$,

$$\exists u, v, w, x, y \; \forall i \; P(z, u, v, w, x, y; i)$$

COMPLEMENT IS

$$\forall u, v, w, x, y \; \exists i \; \neg P(z, u, v, w, x, y; i)$$

WE MOVED i TO START, DOING

$$\exists i \; \forall u, v, w, x, y \; \neg P(z, u, v, w, x, y; i)$$

BUT THAT LIMITS US TO CHOOSING i INDEPENDENT OF u, v, w, x, y

P.S. $P(z, u, v, w, x, y; i) = z = uvwxy$, $|vwx| \leq N$, $|y| > 0$ AND $\forall i \geq 0 \; uv^iwx^iy \in L$

REDO OF MIN

CASE 1:

IF vx CONTAINS NO 'c's THEN IT CONTAINS AT LEAST ON 'b' OR ONE 'a'
CHOOSE $i=0$ AND THEN WE ERASE SOME 'a's AND 'b's (AT LEAST ONE OF ONE OF THESE) BUT NO 'c's, SO WE HAVE MORE 'c''s THAN ONE OF 'a's OR 'b's AND SO $uv^0wx^0y \notin L$

CASE 2:

IF vx CONTAINS A 'c' THEN IT CANNOT CONTAIN ANY a's.
CHOOSE $i=2$ AND THEN WE HAVE MORE THAN N 'c''s BUT JUST N 'a's SO $k \neq \min(i,j)$ - MORE 'c's THAN 'a's AND THEREFORE $uv^2wx^2y \notin L$

CASES 1&2 COVER ALL POSSIBILITIES SO L IS NOT A CFL BY PUMPING LEMMA

WEEK 9

SOLVABLE PROBLEMS
CFLs

$w \in L$?

RUN CKY WITH CNF, G
IF S IN FINAL CELL
$w \in L$

$L = \emptyset$

REDUCE G
IF S NON-PRODUCTIVE
$L = \emptyset$; ELSE $L \neq \emptyset$

L FINITE

REDUCE G
RUN DFS (S)
IF NO LOOPS, L FINITE;
ELSE L INFINITE

CSG

$\alpha \to \beta \qquad |\alpha| \le |\beta|,$

$\alpha \in (V \cup \Sigma)^* V (V \cup \Sigma)^*$

$\beta \in (V \cup \Sigma)^+$

ONE EXCEPTION IS

$S \to \lambda$ IF $\lambda \in L$

AND THEN S CANNOT BE ON RHS

$L = \{a^n b^n c^n \mid n > 0\}$

$G = (\{A \quad\}, \{a, b, c\}, R, A)$

$A \to aBbc \mid abc$

$B \to aBbC \mid abC$

$Cb \to bC$

$Cc \to cc$

$L = \{ww \mid w \in \{0,1\}^+\}$ $\qquad 010 \ldots\ldots \mid \langle 0\rangle$ $\quad 0\langle 1\rangle$ $\quad \langle 0\rangle$

$S \to 00 \mid 11 \mid 0A\langle 0\rangle \mid 1A\langle 1\rangle$

$A \to 0AZ \mid 1AX \mid 0Z \mid 1X$

$\left.\begin{array}{l} Z0 \to 0Z \\ Z1 \to 1Z \\ X0 \to 0X \\ X1 \to 1X \end{array}\right\}$ SHUTTLE

$Z\langle 0\rangle \to 0\langle 0\rangle$

$Z\langle 1\rangle \to 1\langle 0\rangle$

$X\langle 0\rangle \to 0\langle 1\rangle$

$X\langle 1\rangle \to 1\langle 1\rangle$

$\langle 0\rangle \to 0$

$\langle 1\rangle \to 1$

LBA

SIMPLE VIEW
R/W TAPE

q_0 ¢ | w | \$

ACCEPT BY FINAL STATE

ACTIONS ARE
READ WRITE MOVE (LEFT/RIGHT/STAY)

OFTEN EASIEST TO VIEW
OPERATIONS AS BEING ABLE
TO LOOK LEFT OR RIGHT (BASICALLY
MOVES EITHER WAY)

CAN ALSO VIEW AS MULTITRACK
(FINITE # OF TRACKS)
FOR EXAMPLE $(\{¢,\$\} \cup \Sigma) \times (\{¢,\$\} \cup \Gamma)$
AS TAPE ALPHABET WITH CHANNEL
1 HAVING INPUT.

QUICK EXAMPLE OF LBA

$L = \{a^n b^n c^n \mid n > 0\}$

$q_0 ¢ \to ¢ q_1$

$q_1 a \to x q_2$

$q_2 b \to y q_3$

$q_3 c \to q_4 z$

$q_2 a \to a q_2$

$q_3 b \to b q_3$

$z q_4 \to q_4 z$

$q_2 y \to y q_2$

$q_3 z \to z q_3$

$b q_4 \to q_4 b$

$y q_4 \to q_4 y$

$a q_4 \to q_4 a$

$x q_4 \to x q_1$

$q_f z \to z q_f$

$q_1 y \to y q_f$

$q_f y \to y q_f$

$q_f \$ \to \$ q_f$

$$q_0 ¢ a^n b^n c^n \$ \vdash^* ¢ x^k q_1 a^{n-k} y^k b^{n-k} z^k c^{n-k} \$$$

$$\vdash^* ¢ x^n q_1 y^n z^n \$ \vdash^* ¢ x^n y^n z^n \$ q_f$$

TAPE ALPHABET IS

$\{¢, \$, x, y, z, a, b, c\}$

ALGORITHMS AND PROCEDURES

PROCEDURES ARE JUST PROGRAMS OR PROCESSES THAT ARE CLEARLY COMPUTABLE.

PROCEDURES NEED NOT HALT FOR ALL INPUT

ALGORITHMS ARE PROCEDURES THAT HALT ON ALL INPUT

PREDICATES ARE PROCEDURES THAT PRODUCE TRUE/FALSE (YES/NO, 1/0) ANSWERS (OUTPUT)

DECISION PROBLEMS ARE PROBLEMS WHERE EACH INSTANCE HAS A TRUE/FALSE ANSWER

NOTATION:

CONVERGES

$f(x)\downarrow$ MEANS PROCEDURE f HALTS (GIVES AN OUTPUT) WHEN EVALUATED AT x

$f(x)\uparrow$ MEANS PROCEDURE f DIVERGES WHEN EVALUATED AT x

IF f IS AN ALGORITHM THEN $\forall x\, f(x)\downarrow$

SOLVED

UNSOLVED

TEMPORAL

SOLVABLE
(RECURSIVE)
(DECIDABLE)

UNSOLVABLE
(NON-RECURSIVE)
(UNDECIDABLE)

INHERENT

SEMI-DECIDABLE
(ENUMERABLE)
(GENERABLE)
(RECURSIVELY ENUMERABLE)
(RE)

NOT SEMI-DECIDABLE
(NON-ENUMERABLE)
(NON-GENERABLE)
(NON-RE)

INHERENT

COMPUTABILITY

SOLVED IS CONCRETE

SOLVABLE IS EXISTENTIAL

E.G., P=NP? IS SOLVABLE

ANSWER IS "YES" OR "NO"

AND ALGORITHMS EXIST FOR

EACH POSSIBLE ANSWER.

P=NP? IS UNSOLVED AS NO

ONE HAS PROVED THE PROPERTY

TO BE SO, OR HAS REFUTED IT.

$$\nexists\ a, b, c\ [a^n = b^n + c^n]$$

$$a, b, c \in Z^+,\ n > 2$$

EXAMPLES

FERMAT'S LAST THEOREM IS SOLVED AND ALWAYS WAS SOLVABLE

$$\nexists\, n>2, a, b, c\, [a^n + b^n = c^n]$$

WHERE n, a, b, c ARE NATURAL NUMBERS

PROPERTY OF POLYNOMIAL TIME (DECISION) PROBLEMS VERSUS NON-DET POLYNOMIAL TIME ONES

$$P \overset{?}{\neq} NP$$

IS UNSOLVED BUT SOLVABLE

PROPERTY TO DETERMINE, FOR ARBITRARY PROGRAM, P, AND INPUT, x, WHETHER OR NOT $P(x)$ EVENTUALLY HALTS IS UNSOLVABLE BUT SEMI-DECIDABLE

NOTE: $P(x)\downarrow$ MEANS $P(x)$ CONVERGES

MORE EXAMPLES

PROPERTY TO DETERMINE, FOR ARBITRARY PROGRAM P, WHETHER OR NOT

$\forall x \; P(x)\downarrow$ IS UNSOLVABLE AND NOT EVEN SEMI-DECIDABLE

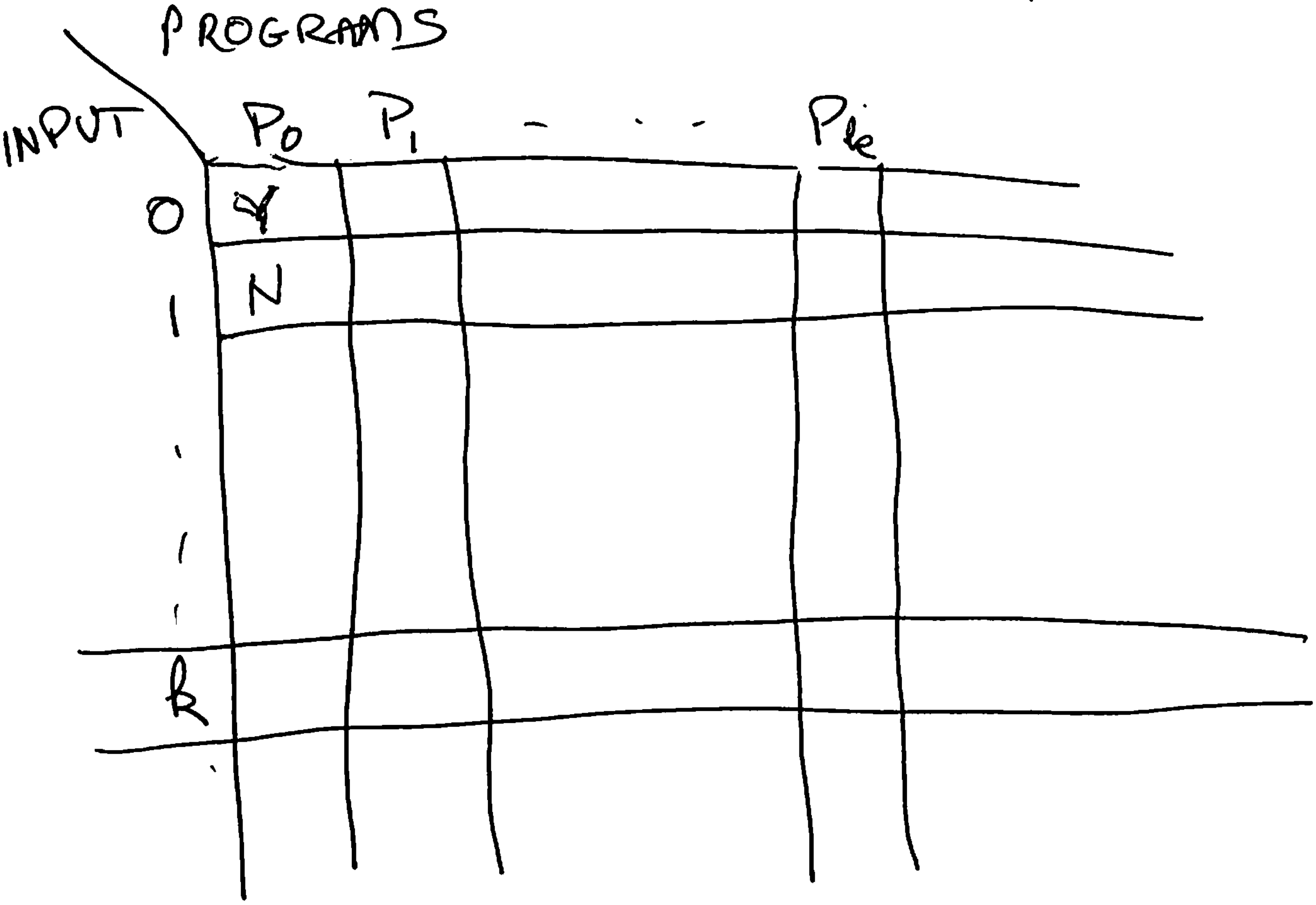
PROPERTY TO DETERMINE, FOR ARBITRARY POLYNOMIAL $P(x_1, \ldots, x_n)$ WHETHER OR NOT $\exists x_1, \ldots, x_n [P(x_1, \ldots, x_n) = 0]$

IS UNSOLVABLE BUT RE (SEMI-DECIDABLE)

CAN DO A HIGHLY STRUCTURED SEARCH (BREADTH FIRST) FOR A SOLUTION

LIMIT ON ALGORITHS TO A COUNTABLE SET MEANS THERE MUST BE SOME THAT CANNOT BE SOLVED OR EVEN ENUMERATED.

LAZY COMPUTATION
HALTING PROBLEM



i, j SLOT SAYS "Y" IF $P_i(j)\downarrow$
SAYS "N" IF $P_i(j)\uparrow$

$P_i(i)\downarrow$ IF i-TH PROGRAM LIKES INPUT i.

~~DEFINE~~ $D(i) = \begin{cases} \overset{\text{LEAST}}{\mu y}[y = y+1] & \text{IF } P_i(i)\downarrow \\ 0 & \text{IF } P_i(i)\uparrow \end{cases}$

D IS A PROGRAM. SAY $P_d = D$ THEN
$D(d)\downarrow$ IFF $P_d(d)\downarrow$ IFF $D(d)\uparrow$

WHAT CONTRADICTION REQUIRED

TO TALK ABOUT $P_0, P_1, \ldots$
WE NEED THAT WE CAN EFFECTIVELY MAP NATURAL NUMBERS TO PROGRAMS

IN CASE OF PROGRAMMING LANUAGES THAT IS JUST SORTING THEM, OR REALLY SORTING SYNTACTICALLY CORRECT PROGRAMS.

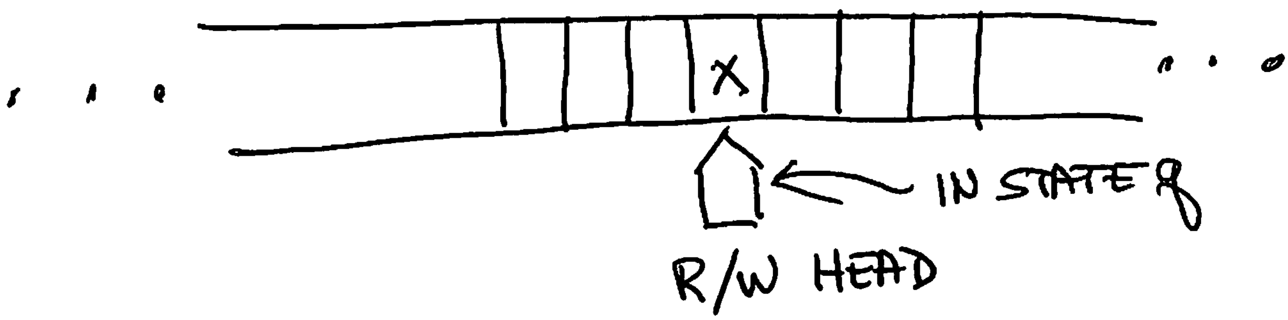
TO USE NOTATION $\mu y [y = y+1]$
WE JUST NEED A CONTROL CONSTRUCT THAT CAN RUN FOREVER.

COULD DEFINE D BY

```
D(i){
    WHILE (HALT(i,i));
    RETURN 0;
}
```

ASSUMING PREDICATE HALT EXISTS WHERE HALT(i,i) IFF $P_i(i)\downarrow$

WEEK 9
TURING MACHINE



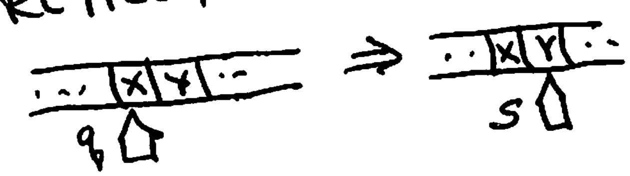
CAN READ CHARACTER IN SCANNED CELL (SAY X) AND BASED ON CURRENT STATE (SAY q) AND X (q, X IS CALLED THE DISCIMINANT)

CAN EITHER
- (a) REWRITE X AS SOME OTHER SYMBOL
- (b) MOVE RIGHT ; OR
- (c) MOVE LEFT

AND THEN CHANGE STATE

NOTE: THIS IS REALLY POST'S NOTATION

q X R s



WEEK 9
TM TAPE

UNMARKED PARTS OF TAPE ARE BLANK
TAPE STARTS WITH INPUT (FINITE)
AT EACH STAGE IT MIGHT MOVE TO PARTS OF TAPE NEVER VISITED BEFORE AND MAY WRITE NEW VALUES.

BASED ON THIS, TAPE IS ALWAYS FINITELY MARKED AND ONLY A FINITE NUMBER OF CELLS CAN BE VISITED IN ANY FINITE PERIOD OF TIME

FOR OUR PURPOSES, WE WILL USE TAPE ALPHABET OF $\{0, 1\}$ AND
BLANK

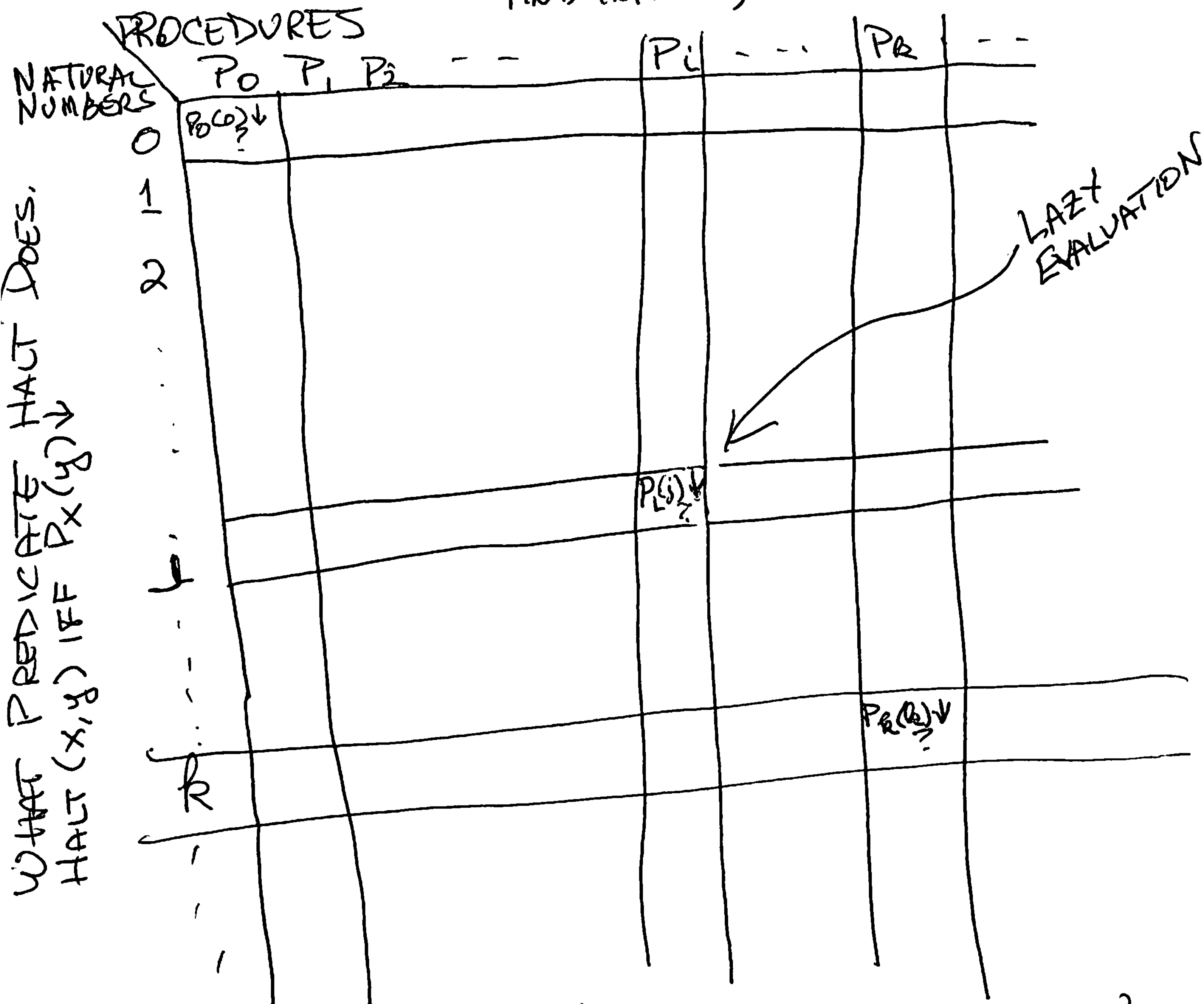
DENOTE NUMBERS IN UNARY.

$M = (Q, \{0,1\}, T)$ ← TABLE OF QUAD MAPPING
$Q \times \{0,1\} \to Q \times \{0,1,R,L\}$

HALTING IS JUST ENTERING A STATE q WITH INPUT x, WHERE qx HAS NO ENTRY IN T.

HALTING PROBLEM

ASSUME WE CAN DECIDE FOR ARBITRARY PROCEDURE, P, AND INPUT X, WHETHER OR NOT $P(X)\downarrow$

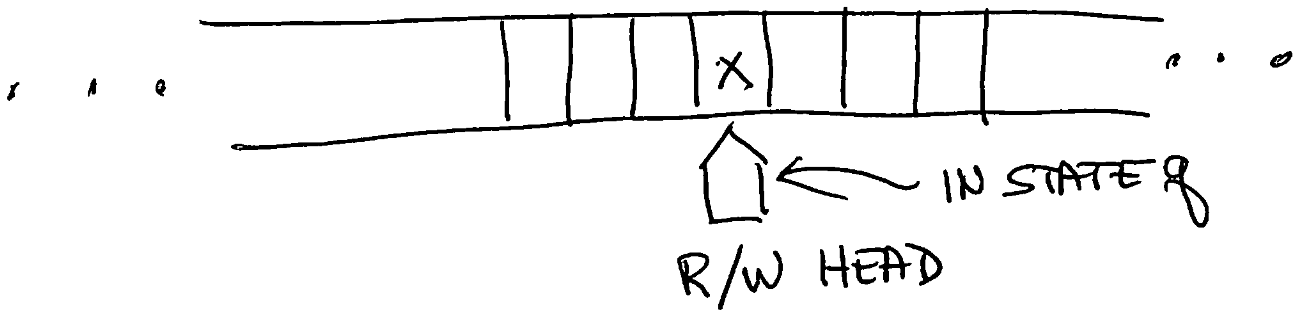


DEFINE $D(x) = \{\text{WHILE } P_x(x);\ \text{RETURN}(1);\}$
$= \{\text{WHILE HALT}(x,x);\ \text{RETURN}(1);\}$

SINCE HALT IS AN ALGORITHM, IT IS A PROCEDURE AND HENCE SO IS D. ASSUME $D = P_d$ THEN

$D(d)\downarrow$ IFF $P_d(d)\downarrow$ IFF HALT(d,d)=TRUE IFF
$P_d(d)\uparrow$ IFF $D(d)\uparrow$

TURING MACHINE



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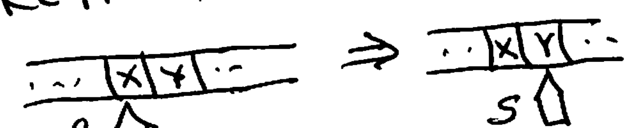

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