PUMPING LEMMA FOR CFLs

IF L IS A CFL, THEN $\exists N>0$, SUCH THAT IF $z \in L$ AND $|z| \geq N$ THEN $z = uvwxy$, WHERE $|vwx| \leq N$, $|vx|>0$ AND $\forall i \geq 0$ $uv^iwx^iy \in L$

NOTE THAT IF $|\Sigma|=1$ THEN PUMPING LEMMA FOR CFL DEGENERATES TO PUMPING LEMMA FOR REGULAR

THUS, IF $L \subseteq \Sigma^*$, $|\Sigma|=1$, AND L IS NOT REGULAR THEN L IS NOT A CFL

$L = \{ww \mid w \in \{a,b\}^*\}$ IS NOT A CFL

1. ASSUME L IS A CFL
2. PL GIVES YOU N
3. YOU CHOOSE $a^N b^N a^N b^N$
4. PL SPLITS $a^N b^N a^N b^N$ INTO $uvwxy$
   WHERE $|vwx| \leq N$, $|vx| > 0$
   AND STATES $\forall i \geq 0$ $uv^iwx^iy \in L$
5. a.) IF vwx IS OVER a'S ONLY
   THEN SINCE $|vwx| \leq N$
   vwx CANNOT SPAN FROM ONE OF
   THE a^N SUBSTRINGS TO OTHER,
   AND SO FOR $i=0$, WE GET EITHER
   $a^{N-|vx|} b^N a^N b^N \in L$ OR $a^N b^N a^{N-|vx|} b^N \in L$
   AND BOTH ARE NOT SO.
   b) IF vwx IS OVER b'S ONLY, THE
   SAME ARGUMENTS APPLY
   c) IF vwx SPANS a'S AND b'S THEN
   THIS MUST BE WITHIN ONE OF
   THE $a^N b^N$ OR $b^N a^N$ SUBSTRINGS
   AS $|vwx| \leq N$. IF $i=0$ THEN WE
   REDUCE SOME a'S AND SOME b'S, BUT NOT ALL,
   WITHIN JUST ONE OF THE CONSECUTIVE
   a'S AND b'S, BUT NOT THE OTHER
   AND, SO THE RESULTING STRING IS NOT
   $\in L$

WEEK 8
PUSHDOWN AUTOMATA
PDA

$$A = (Q, \Sigma, \Gamma, \delta, q_0, Z_0, F)$$

- STATES (FINITE)
- LANG. ALPHABET (FINITE)
- STACK SYMBOLS (FINITE
- START STATE
- OPTIONAL BOTTOM OF STACKMARKER
- OPTIONAL FINAL STATES

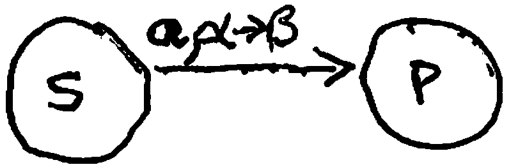
TRANSITION FUNCTION

$$\delta : Q \times \Sigma_e \times \Gamma_e \to 2^{Q \times \Gamma^*}$$

CAN EXTEND TO Γ^* CAN LIMIT TO Γ_e BY GROWING STATES

PDA DIAGRAMS

SOME TEXTS USE $a, X / Y$ WHICH IS FINE ALSO



$$\delta(s, a, \alpha) \supseteq \{(p, \beta)\}$$

PDA LANGUAGES $\equiv$ CFLs

INSTANTANEOUS DESCRIPTIONS (ID)

$[q, w, \gamma]$ — q - CURRENT STATE
w - REMAINING INPUT
γ - STACK CONTENTS READ LEFT (TOP) TO RIGHT (BOTTOM)

SINGLE STEP

$[q, ax, Z\alpha] \vdash [p, x, \beta\alpha]$ IF $\delta(q, a, Z) \ni (p, \beta)$

MULTISTEP $\vdash^*$ REFLEXIVE TRANSITIVE CLOSURE OF $\vdash$

GIVEN $A = (Q, \Sigma, \Gamma, \delta, q_0, Z_0, F)$
THERE CAN BE THREE NOTIONS OF ACCEPTANCE

FINAL STATE

$\mathcal{L}(A) = \{w \mid [q_0, w, Z_0] \vdash^* [f, \lambda, \beta]\}, f \in F$

EMPTY STACK

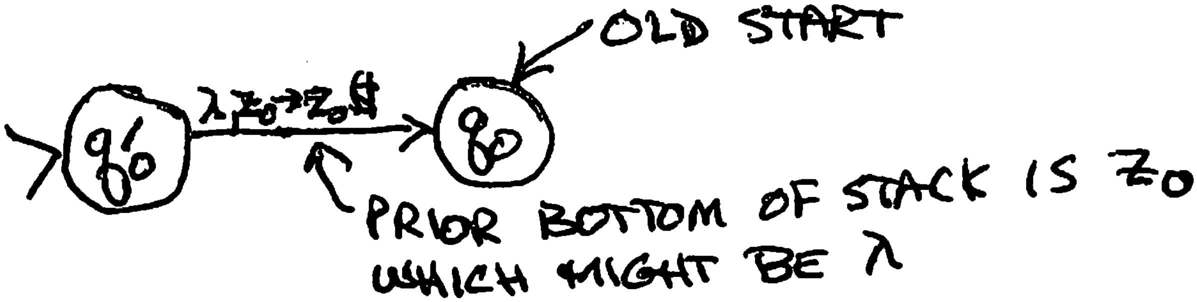
$N(A) = \{w \mid [q_0, w, Z_0] \vdash^* [q, \lambda, \lambda]\}, q \in Q$

EMPTY STACK AND FINAL STATE

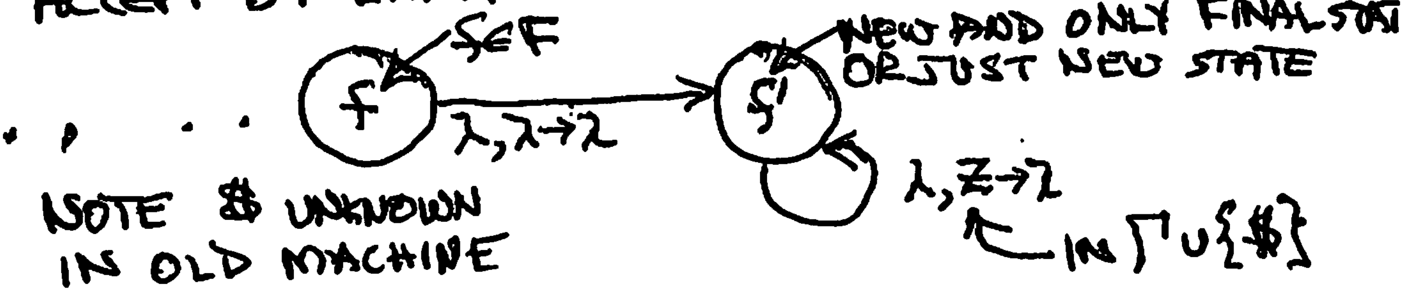
$E(A) = \{w \mid [q_0, w, Z_0] \vdash^* [f, \lambda, \lambda]\}, f \in F$

EQUIVALENCY OF LANGUAGE CLASSES, $\mathcal{L}(A)$, $N(A)$, $E(A)$, WHERE A RANGES OVER ALL PDAs

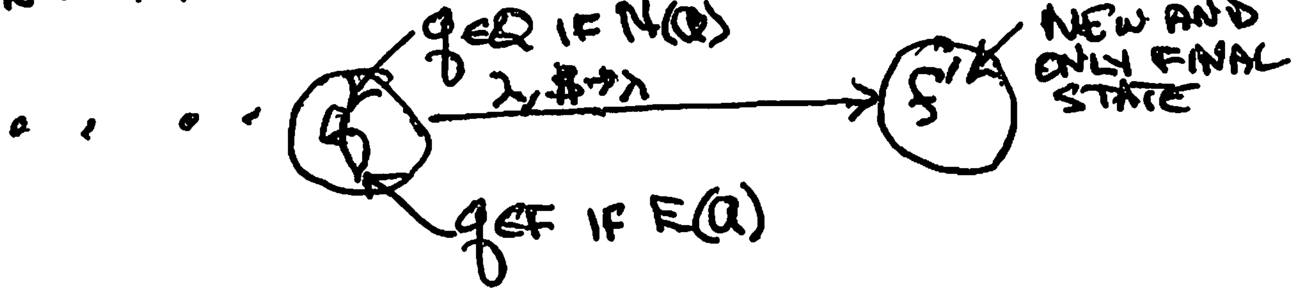
- CONVERTING ONE FORM TO ANOTHER
  FOR EACH CASE, ASSUME q_0' IS A NEW STATE (NEW START) AND \$ IS A NEW STACK SYMBOL PLACED ON BOTTOM OF STACK. FOR ALL CASES, START WITH



- CHANGE ACCEPT BY FINAL STATE TO ACCEPT BY EMPTY STACK OR EMPTY AND FINAL STATE



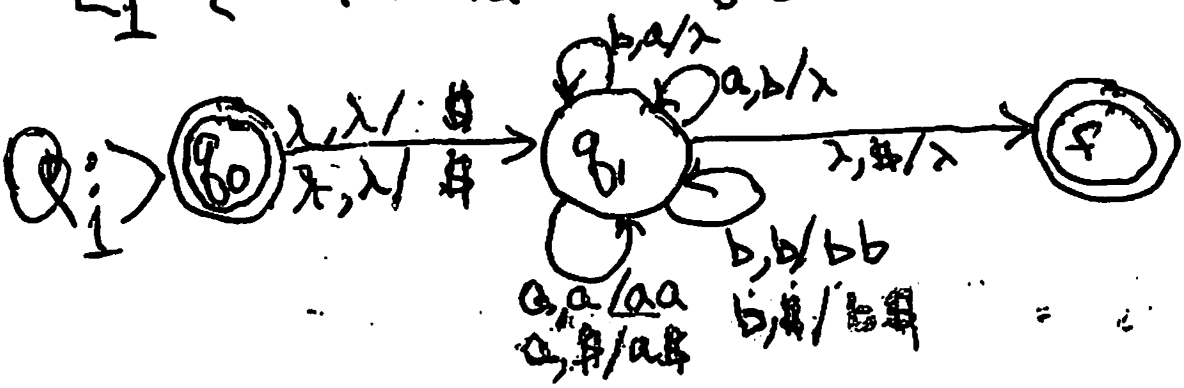
- CHANGE ACCEPT BY EMPTY STACK OR EMPTY AND FINAL STATE TO ACCEPT BY FINAL STATE



101. WEEK 8 (4)

EXAMPLE PDA

$L_1 = \{ w \mid |w|_a = |w|_b \}$. ASSUME λ ON STACK



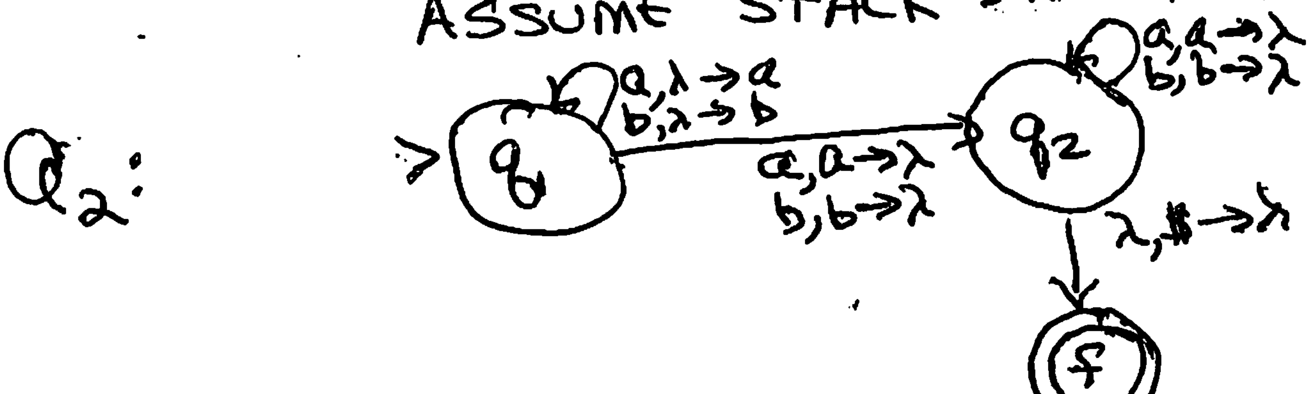
I USED / WHERE BOOK USES $\rightarrow$

THIS GETS $L_1 = E(Q_1)$

THIS IS NON-DETERMINISTIC

$L_2 = \{ w w^R \mid w \in \{a, b\}^+ \}$

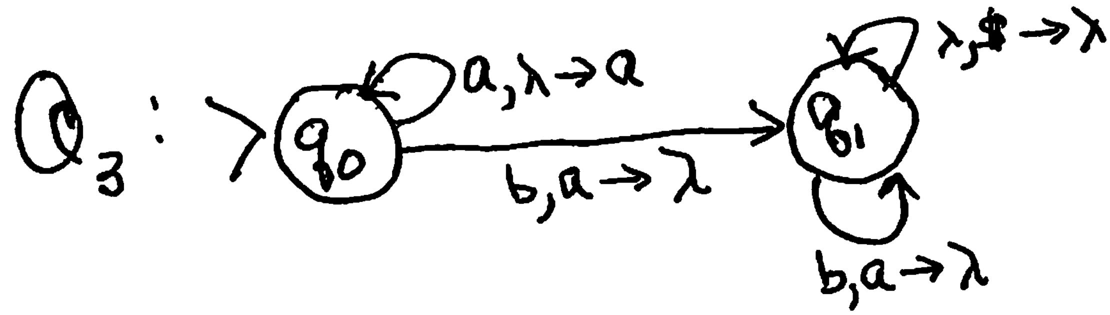
ASSUME STACK STARTS WITH \$



THIS GETS $L_2 = \mathcal{L}(Q_2) = E(Q_2)$

THIS IS ALSO NON-DETERMINISTIC

$L_3 = \{a^n b^n \mid n > 0\}$. ASSUME \$ ON STACK



THIS GETS $L_3 = N(Q_3)$

THIS IS DETERMINSTIC

DETERMINISM FOR PDAs

1) FOR EACH $q \in Q$ & $z \in \Gamma$ & $a \in \Sigma$
   IF $|\delta(q, \lambda, z)| > 0$ THEN $|\delta(q, a, z)| = 0$

2) FOR NO $q \in Q$, $z \in \Gamma$ & $a \in \Sigma_\epsilon$
   IS $|\delta(q, a, z)| > 1$

WEEK 8

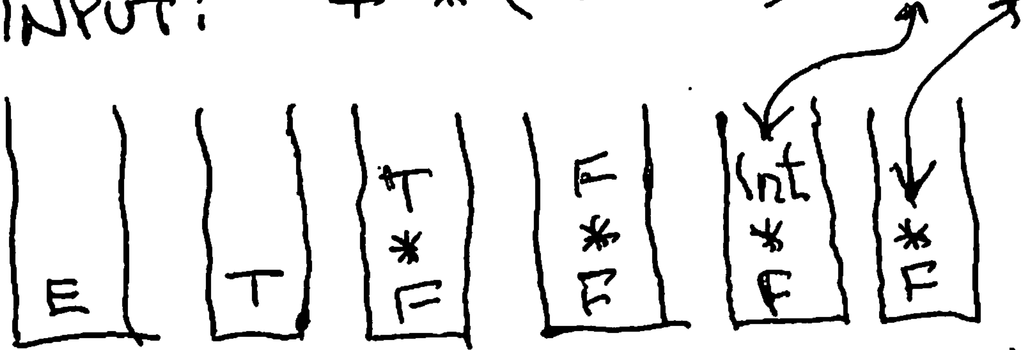
TOP-DOWN PARSER

$E \to E+T \mid T$

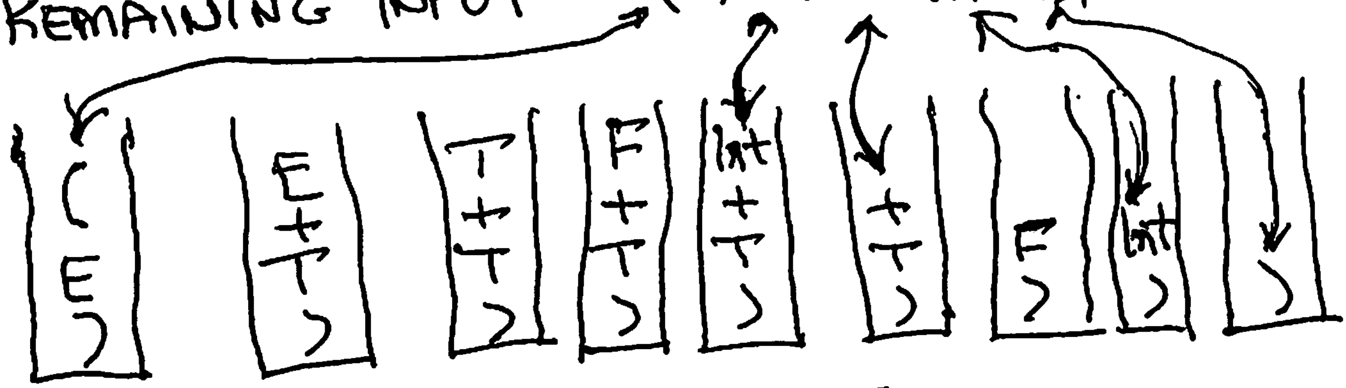
$T \to T*F \mid F$

$F \to (E) \mid int$

INPUT: $7 * (3+21) \Rightarrow int * (int + int)$



REMAINING INPUT: $(int + int)$



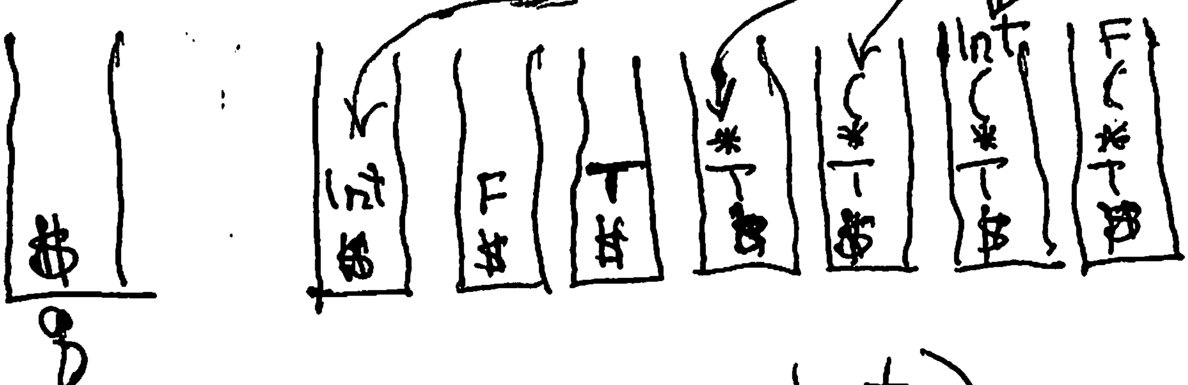
ACCEPTS BY EMPTY STACK

WEEK 8

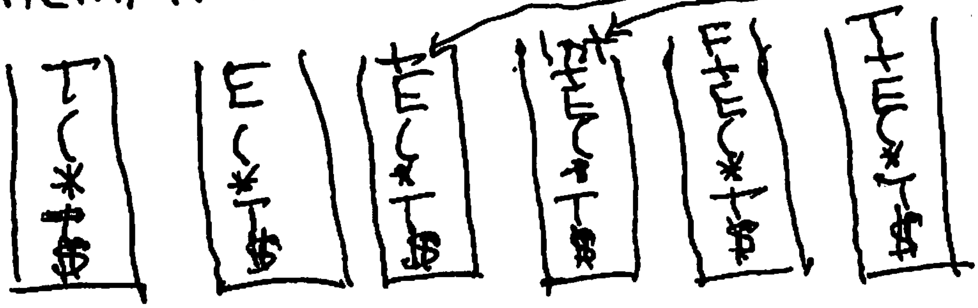
BOTTOM-UP PARSER

$E \rightarrow E+T \mid T$
$T \rightarrow T*F \mid F$
$F \rightarrow (E) \mid \text{Int}$

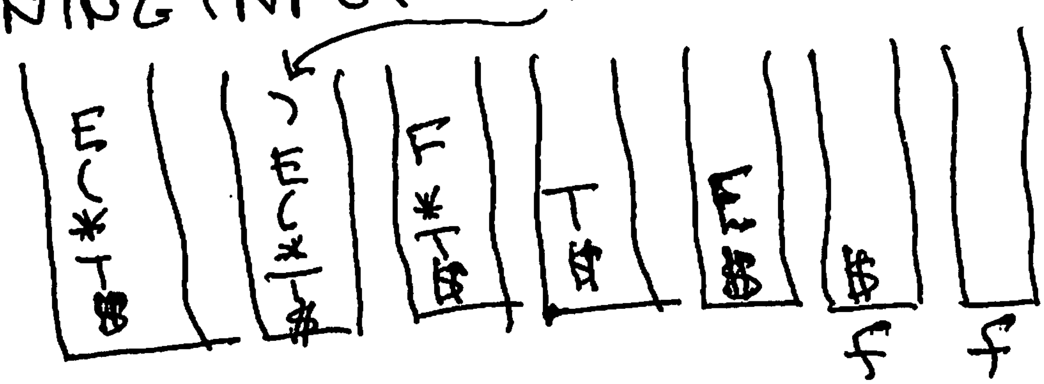
INPUT: $7*(3+21) \Rightarrow \text{Int}*(\text{Int}+\text{Int})$



REMAINING INPUT + Int)



REMAINING INPUT)



ACCEPTS BY FINAL STATE AND EMPTY STATE
OR JUST EMPTY STACK (CAN ALTER FOR FINAL)

LIMITING PDA TO PUSH/POP

PUSH: PUSH(α) IS EQUIVALENT TO

$$\delta(q, a, z) \supseteq \{(p, \alpha z)\}$$

WHERE WE JUST USE

$$\delta(q, a, z) \supseteq \{(p, \text{PUSH}(\alpha))\}$$

POP: POP IS EQUIVALENT TO

$$\delta(q, a, z) \supseteq \{(p, \lambda)\}$$

WHERE WE JUST USE

$$\delta(q, a, z) \supseteq \{(p, \text{POP})\}$$

IF WANT TO SIMULATE STANDARD OPERATION OF

$$\delta(q, a, z) \supseteq \{(p, \alpha)\}$$

CAN DO

$$\delta(q, a, z) \supseteq \{(p', \text{POP})\}$$

$$\delta(p', \lambda, X) \supseteq \{(p, \text{PUSH}(\alpha))\}$$

ANY ELEMENT OF Γ OR λ IF ALLOWED

$$[q_0, w, \$] \vdash^* [f, \lambda, \lambda]$$

PDA TO CFG

$Q = (Q, \Sigma, \Gamma, \delta, q_0, \emptyset, \{f\})$

LIMITED TO PUSH/POP

TECHNIQUE # 1:

NON-TERMINALS ARE OF FORM

$A_{t,u}$

GOAL IS $A_{t,u} \overset{*}{\Rightarrow} w \in \Sigma^*$

WHEN $[t, w, \alpha] \vdash^* [u, \lambda, \alpha]$

START IS

$A_{q_0, f}$

RULES ARE

$A_{q,q} \to \lambda$ $\quad \forall q \in Q$ REFLEXIVE

$A_{t,u} \to A_{t,v} A_{v,u}$ TRANSITIVE

AND

$A_{t,u} \to a A_{r,s} b$

WHEN $\delta(t, a, \lambda) \ni \{(r, x)\}$ PUSH

& $\delta(s, b, x) \ni \{(u, \lambda)\}$ POP

WEEK 8
PDA TO CFG

$Q = (Q, \Sigma, \Gamma, \delta, q_0, z, F)$ — $\{f\}$

USED IN TECHNIQUE 1 $z = \$$; NOT USED IN BOOK'S APPROACH SO $\emptyset$

TECHNIQUE #2:

LIMITED TO PUSH + POP

NON-TERMINALS ARE OF FORM

$\langle p, z, q \rangle$

GOAL IS $\langle p, z, q \rangle \stackrel{*}{\Rightarrow} w \in \Sigma^*$

WHEN $[p, w, z] \vdash^* [q, \lambda, \lambda]$

START SYMBOL IS

$S \to \langle q_0, \$, f \rangle$

CAN ACTUALLY DO FOR EMPTY STACK ONLY BY HAVING

$S \to \langle q_0, \$, q \rangle \quad \forall q \in Q$

RULES, OTHER THAN START, ARE

$\langle q, x, p \rangle \to a \langle s, y, t \rangle \langle t, x, p \rangle$

WHENEVER $\delta(q, a, x) \supseteq \{(s, \text{PUSH}(y))\}$

$\langle q, x, p \rangle \to a$

WHENEVER $\delta(q, a, x) \supseteq \{(p, \text{POP})\}$

GOAL: $\langle q_0, \$, f \rangle \stackrel{*}{\Rightarrow} w$, WHENEVER $w \in L(Q)$

GREIBACH NORMAL FORM

ALL RULES, EXCEPT PERHAPS, $S \to \lambda$

LIMITED TO

$$A \to a\alpha \qquad A \in V, a \in \Sigma, \alpha \in V^*$$

PROVIDES LINEAR PARSE IF

WE CAN AVOID

$$\left.\begin{array}{l}\text{SHIFT/REDUCE} \\ \& \\ \text{REDUCE/REDUCE}\end{array}\right\} \text{CONFLICTS}$$