COT 4210 Spring 2017 **Midterm 2** **Name: ________________**
Total Points Available 67 **Your Raw Score _______** **Grade:____________**

2 ~~**1.** Let **A = ({q1, … q10}, {0,1}, q1, {q7})** be some DFA. Assume you have computed the sets, $R^{k}_{i,j}$, for **0 ≤ k ≤ 9, 1 ≤ i ≤ 10, 1, 1 ≤ j ≤10**. How do you compute **L(A) =** $R^{10}_{1,7}$, based on the previously computed values of the $R^{k}_{i,j}$'s?~~

I WON'T BE ASKING THIS TYPE OF QUESTION BECAUSE THE CLASS DID WELL ON IT IN EXAM#!

4 **2.** Write a Context Free Grammar for the language **L**, where
$L = \{ a^i b^j c^k \mid i \le (j + k) \}$

5 **3.** Assume **A** and **B** are arbitrary Context Free languages. Indicate, for each of the following operations, whether the language **L** is guaranteed to be Context Free (Note: Regular languages are Context Free). No proofs or examples are required.

Operation	**Is L guaranteed to be a CFL? (Y or N)**
L ⊂ A (Subset)	
L = A ∩ B (Intersection)	
L = A • B (Concatenation)	
L = A ⊕ B ({ x | x is in either A or B, but not both }	
L = Max(A) ({ x | x∈A but no xy∈A, |y|>0 }	

8 **4.** Use the Pumping Lemma for CFLs to show that the following language **L** is not Context Free.
$\mathbf{L = \{ a^n b^{2^n} \mid n>0 \}}$. Be explicit as to why each case you analyze fails to be an instance of **L**.
I will do the first two steps for you.

ME: Assume L is Context Free
PL: Provides a whole number N>0 that is the value associated with L based on the Pumping Lemma

6 **5.** Consider some languages **A** and **B** that are both Context Free, and neither is Regular. Define **L = A ∪ B**. Give explicit examples of languages **A** and **B**, and explicitly describe **L**, or argue that this is impossible based on some well-known result, for each of the following.

a.) **L** is Regular

b.) **L** is Context Free, non-Regular.

c.) **L** is Context Sensitive, non-Context-Free.

10 **6.** Present the **CKY** recognition matrix for the string **abbccc** assuming the Chomsky Normal Form grammar, **G = ({S, A, B, C, X, Y, Z }, {a,b,c}, R, S)**, specified by the rules **R**: Note: **abbccc** is in **L(G)** so that should help you if you make an error and don't see **S** at bottom of matrix.

S $\rightarrow$ AB
A $\rightarrow$ XA | a
B $\rightarrow$ CZ | BZ | b | c
C $\rightarrow$ YB
X $\rightarrow$ a
Y $\rightarrow$ b
Z $\rightarrow$ c

	a	**b**	**b**	**c**	**c**	**c**
1						
2						
3						
4						
5						
6						

A little help from your friends

Non-Terminal	**First Symbol in Rules**	**Second Symbol in Rules**
S	None	None
A	**S $\rightarrow$ AB**	**A $\rightarrow$ XA**
B	**B $\rightarrow$ BZ**	**S $\rightarrow$ AB; C $\rightarrow$ YB**
C	**B $\rightarrow$ CZ**	None
X	**A $\rightarrow$ XA**	None
Y	**C $\rightarrow$ YB**	None
Z	None	**B $\rightarrow$ BZ; B $\rightarrow$ CZ**

8 7. Prove that Context-Free Languages are closed under **div3** where **L** is a CFL over the alphabet **Σ**, and **div3(L) = { x | xy ∈ L** and **|x| modulo 3 = 0** and **|y| ∈ {0,1,2} }.**
In words, we remove as few characters as needed from the end of a string in **L**, so the resulting string's length is a multiple of **3**.
You may assume substitution **f(a) = {a, a'},** and homomorphisms **g(a) = a'** and **h(a) = a, h(a') = λ**.
Here **a∈Σ** and **a'** is a distinct new character associated with each **a∈Σ**.
You must be very explicit, describing what is produced by each transformation you apply and what kind of language results.

12 **8.** Consider the CFG **G = ({ S, A, B } , { a , b } , R , S)** where **R** is:

S → AB
A → aA | a
B → aBb | ab

In the PDAs below, you may (and are encouraged) to use a transition diagram where transitions have arcs with labels of form **a, α → β** where **a** ∈ Σ∪{λ}, α, β ∈ Γ*. Note: This just means that you can use extended stack operations that push more than one symbol onto stack.

a.) Present a pushdown automaton that parses the language L(G) using a top down strategy.
INITIAL CONTENTS OF STACK = ______

b.) Now, using the notation of **ID**s (Instantaneous Descriptions, **[q, x, z]**), describe how your PDA in (a) accepts strings generated by **G**.

c.) Present a pushdown automaton that parses the language **L(G)** using a bottom up strategy. Note: I am fine with your showing strings that are on top of the stack in either reversed or non-reversed form.
INITIAL CONTENTS OF STACK = ______

d) Now, using the notation of **ID**s (Instantaneous Descriptions, **[q, x, z]**), describe how your PDA in (c) accepts strings generated by **G**.

9. Consider the context-free grammar **G = ({ S, A, B, C, D }, { a,b,c }, R, S)**, where **R** is:

S → BC | AC | ABC
A → aA | λ
B → ABb | Bb
C → bCc | bc
D → bBc | Dc | λ

3 **a.)** Remove λ-rules, creating an equivalent grammar **G'**. Show all rules. ***Nullable* = { }**

2 **b.)** Remove all **unit** rules', creating an equivalent grammar **G''**. Show all rules.
Unit(S) = { }; Unit(A) = { }; Unit(B) = { }; Unit (C) = { }; Unit(D) = { }

2 **c.)** Remove all unproductive symbols, creating an equivalent grammar **G'''**. Show all rules.
Productive = { }; Unproductive = { }

2 **d.)** Remove all unreachable symbols, creating an equivalent grammar $\mathbf{G^{iv}}$. Show all rules.
Unreachable = { }

3 **e.)** Convert grammar $\mathbf{G^{iv}}$ to its **Chomsky Normal Form** equivalent, $\mathbf{G^{V}}$. Show all rules.