COT 4210 Spring 2017 Sample Midterm 2 Name: ______

1. Write a **Context Free Grammar** for the language
   $\mathbf{L = \{ a^k\ b^m\ c^n \mid k = n + m, \text{ or } m = k + n, \text{ or } n = k + m, k>0, m>0, n>0 \}}$.

2. Consider the language
   $\mathbf{L = \{ a^n\ b^{n!} \mid n>0 \}}$.
   Use the Pumping Lemma for Context-Free Languages to show that **L** is not context-free.

3. Present the **CKY** recognition matrix for the string **bbabb** assuming the Chomsky Normal Form grammar, **G = ({S,A,B,C,D }, {a,b}, R, S)**, specified by the rules **R**:

   S → AB | BA | BD
   A → CS | CD | a
   B → DS | b
   C → a
   D → b

	b	b	a	b	b
1					
2					
3					
4					
5					

4. Choosing from among **(Y) yes**, **(N) No**, categorize each of the following closure properties. No proofs are required.

Problem / Language Class (C)	**Regular**	**Context Free**
Closed under union with Context Free languages?		
Closed under quotient with languages of its own class (C), i.e., L1/L2		
Closed under difference with languages of its own class (C), i.e., (difference (L1, L2) = L1 – L2)?		
Closed under intersection with Closed with languages of its own class?		

5. Prove that any class of languages, ***C***, closed under union, concatenation, intersection with regular languages, homomorphism and substitution (e.g., the Context-Free Languages) is closed under **Erase Middle with Regular Sets (em)**, where **L** $\in$ ***C***, **R** is Regular, **L** and **R** are over the alphabet **Σ**, and **L em R = { xz | x,z** $\in$ **Σ**$^+$ and **∃y** $\in$ **R,** such that **xyz** $\in$ **L }.** You may assume substitution **f(a) = {a, a'},** and homomorphisms **g(a) = a'** and **h(a) = a, h(a') = λ**. Here **a∈Σ** and **a'** is a distinct new character associated with each **a∈Σ**.
You must be very explicit, describing what is produced by each transformation you apply.

6. Consider the CFG **G = ({ S, T }, { a, b }, R, S)** where **R** is:

S → a T T | T S | a
T → b S T | b

a.) Present a pushdown automaton that accepts the language generated by this grammar. You may (and are encouraged) to use a transition diagram where transitions have arcs with labels of form **a,** $\alpha \rightarrow \beta$ where **a** $\in \Sigma\cup\{\lambda\}$, $\alpha, \beta \in \Gamma^*$. Note: I am encouraging you to use extended stack operations.

What parsing technique are you using? (Circle one) **top-down** or **bottom-up**
How does your PDA accept? (Circle one) **final state** or **empty stack** or **final state and empty stack**
What is the **initial state**? __________________________
What is the **initial stack content**? __________________________
What are your **final states** (if any)? __________________________

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b.) Now, using the notation of **ID**s (Instantaneous Descriptions, **[q, x, z]**), describe how your PDA accepts strings generated by **G**.

7. Consider the context-free grammar **G = ({ S, A, B }, { a,b }, R, S)**, where **R** is:

S → SAB | BA
A → AB | a
B → bS | b | λ

a.) Remove all **λ**-rules from **G**, creating an equivalent grammar **G**'. Show all rules.
Nullable = { }
G':

b.) Remove all **unit** rules from **G'**, creating an equivalent grammar **G**''. Show all rules.
Unit(S)=Chain(S)={ }; Unit(A)={ }; Unit(B)={ }
G'':

c.) Convert grammar **G''** to its **Chomsky Normal Form** equivalent, **G'''**. Show all rules.
G''':

In exam I may have some Unproductive non-terminals and some Unreachable ones.