COT 4210 Spring 2017 Sample Midterm 2 Key Name: KEY

1. Write a **Context Free Grammar** for the language
$L = \{ a^k\ b^m\ c^n \mid k = n + m,$ or $m = k + n,$ or $n = k + m, k>0, m>0, n>0 \}$.

S → aAc | aA'bbC'c
A → aAc | aA'b | bC'c
A' → aA'b | λ
C' → bC'c | λ

2. Consider the language
$L = \{ a^n\ b^{n!} \mid n>0 \}$.
Use the Pumping Lemma for Context-Free Languages to show that **L** is not context-free.

PL: Provides N>0
We: Choose $a^N b^{N!} \in L$
PL: Splits $a^N b^{N!}$ into uvwxy, $|vwx| \le N$, $|vx| > 0$, such that $\forall i \ge 0\ uv^iwx^iy \in L$
We: Choose i=2
Case 1: vwx contains only b's, then we are increasing the number of b's while leaving the number of a's unchanged. In this case uv^2wx^2y is of form $a^N b^{N!+c}$, c>0 and this is not in L.
Case 2: vwx contains some a's and maybe some b's. Under this circumstances uv^2wx^2y has at least N+1 a's and at most N!+N-1 b's. But $(N+1)! = N!(N+1) = N!*N+N \ge N! + N > N!+N-1$ and so is not in L.
Cases 1 and 2 cover all possible situations, so L is not a CFL.

3. Present the **CKY** recognition matrix for the string **bbabb** assuming the Chomsky Normal Form grammar, **G = ({S,A,B,C,D }, {a,b}, R, S)**, specified by the rules **R**:

S → AB | BA | BD
A → CS | CD | a
B → DS | b
C → a
D → b

	b	b	a	b	b
1	**BD**	**BD**	**AC**	**BD**	**BD**
2	**S**	**S**	**SA**	**S**	
3	**B**	**SB**	**SA**		
4	**SB**	**SB**			
5	**SB**				

4. Choosing from among **(Y) yes**, **(N) No**, categorize each of the following closure properties. No proofs are required.

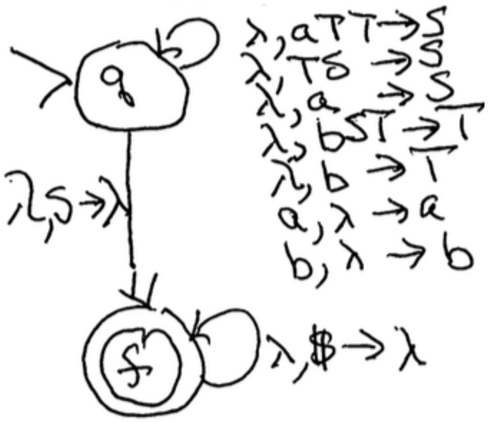
Problem / Language Class (C)	**Regular**	**Context Free**
Closed under union with Context Free languages?	N	Y
Closed under quotient with languages of its own class (C), i.e., L1/L2	Y	N
Closed under difference with languages of its own class (C), i.e., (difference (L1, L2) = L1 – L2)?	Y	N
Closed under intersection with languages of its own class?	Y	N

5. Prove that any class of languages, ***C***, closed under union, concatenation, intersection with regular languages, homomorphism and substitution (e.g., the Context-Free Languages) is closed under **Erase Middle with Regular Sets (em)**, where $\mathbf{L} \in C$, **R** is Regular, **L** and **R** are over the alphabet Σ, and $\mathbf{L\ em\ R} = \{\ \mathbf{xz} \mid \mathbf{x,z} \in \Sigma^+$ and $\exists \mathbf{y} \in \mathbf{R}$, such that $\mathbf{xyz} \in \mathbf{L}\ \}$. You may assume substitution $\mathbf{f(a) = \{a, a'\}}$, and homomorphisms $\mathbf{g(a) = a'}$ and $\mathbf{h(a) = a, h(a') = \lambda}$. Here $\mathbf{a} \in \Sigma$ and **a'** is a distinct new character associated with each $\mathbf{a} \in \Sigma$.
You must be very explicit, describing what is produced by each transformation you apply.

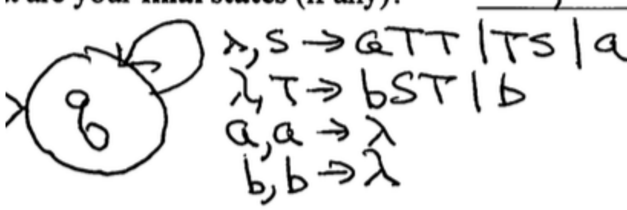
***L em R** = $h(f(L) \cap \Sigma^+ g(R)\ \Sigma^+\ )$*
$f(L) = \{\ \underline{w} \mid w \in L\ \}$ where $\underline{w}$ has some (or none) of its letters primed. f(L) is a CFL since CFLs are closed under substitution.
$g(R) = \{\ y' \mid y \in R\ \}$ where y' has all of its letter primed. g(R) is Regular since Regular languages are closed under homomorphism.
$\Sigma^+ g(R)\ \Sigma^+ = \{\ xy'z \mid x,z \in \Sigma^+$ and $y \in R$, This is a Regular language since Regular languages are closed under concatenation.
$f(L) \cap \Sigma^+ g(R)\ \Sigma^+ = \{\ xy'z \mid xyz \in L$ and $y \in R\}$. This is a CFL since CFLs are closed under intersection with Regular.
L em R = $h(f(L) \cap \Sigma^+ g(R)\ \Sigma^+ = \{\ xz \mid \exists y \in R$ where $xyz \in L\ \}$ is a CFL since CFLs are closed under homomorphism.

6. Consider the CFG **G = ({ S, T }, { a, b }, R, S)** where **R** is:
S → a T T | T S | a
T → b S T | b

a.) Present a pushdown automaton that accepts the language generated by this grammar. You may (and are encouraged) to use a transition diagram where transitions have arcs with labels of form **a, α → β** where **a** ∈ Σ∪{λ}, **α, β** ∈ **Γ***. Note: I am encouraging you to use extended stack operations.



What parsing technique are you using? (Circle one) **top-down** or **bottom-up**
How does your PDA accept? (Circle one) **final state** or **empty stack** or **final state and empty stack**
What is the **initial state**? q
What is the **initial stack content**? $
What are your **final states** (if any)? f



Or can have **a,S → TT | λ; λ,S → TS**
b,T → ST | λ

What parsing technique are you using? (Circle one) **<u>top-down</u>** or **bottom-up**
How does your PDA accept? (Circle one) **final state** or **<u>empty stack</u>** or **final state and empty stack**
What is the **initial state**? ***q***
What is the **initial stack content**? ***S***
What are your **final states** (if any)? ***None***

b.) Now, using the notation of **ID**s (Instantaneous Descriptions, **[q, x, z]**), describe how your PDA accepts strings generated by **G**.

[q, w, $] ⇒* [f, λ, λ] if by final state and empty stack (my solution on (a) Bottom-Up)

[q, w, S] ⇒* [q, λ, λ] if by empty stack (my solution on (a) Top-Down)

7. Consider the context-free grammar $\mathbf{G = (\{ S, A, B \}, \{ a,b \}, R, S)}$, where **R** is:

$\mathbf{S \to SAB \mid BA}$
$\mathbf{A \to AB \mid a}$
$\mathbf{B \to bS \mid b \mid \lambda}$

a.) Remove all λ-rules from **G**, creating an equivalent grammar **G'**. Show all rules.
Nullable = {B}
G':
$S \to SAB \mid SA \mid BA \mid A$
$A \to AB \mid a$
$B \to bS \mid b$

b.) Remove all **unit** rules from **G'**, creating an equivalent grammar **G''**. Show all rules.
Unit(S)=Chain(S)={S,A}; Unit(A)={A}; Unit(B)={B}
G'':
$S \to SAB \mid SA \mid BA \mid AB \mid a$
$A \to AB \mid a$
$B \to bS \mid b$

c.) Convert grammar **G''** to its **Chomsky Normal Form** equivalent, **G'''**. Show all rules.
G''':
$S \to S<AB> \mid SA \mid BA \mid AB \mid a$
$A \to AB \mid a$
$B \to <b>S \mid b$
$<AB> \to AB$
$<b> \to b$

In exam I may have some Unproductive non-terminals and some Unreachable ones.