Assignment # 9.1a Key

1. Use quantification of an algorithmic predicate to estimate the complexity (decidable, re, co-re, non-re) of each of the following, (a)-(d):

a){ f | f is a Fibonacci function, i.e. f(0)=f(1)=1 and f(x+2)=f(x)+f(x+1) }

$\forall x \exists t$[STP(f,x,t) & STP(f,x+1,t) & STP(f,x+2,t) &
(x=0 $\Rightarrow$ VALUE(f,x,t)=0 & x=1 $\Rightarrow$ VALUE(f,x,t)=1 &
x>1 $\Rightarrow$ (VALUE(f,x+2,t) = VALUE(f,x,t)+Value(x+2,t))]

Non-re, Non co-re

Assignment # 9.1b Key

b) (f | |range(f)| > 1 }

∃<x,y,t> [STP(f,x,t) & STP(f,y,t) & (VALUE(f,x,t) ≠ (VALUE(f,y,t))]

RE

Assignment # 9.1c Key

c) { <f,x> | if f(x) converges, it does so in more than (2^x) units of time}

~STP(f, x,2^x)
REC / DEC

Assignment # 9.1d Key

d) { f | f(x) = f(x+1) for at least one value of x }

∃<x,t> [STP(f,x,t) & STP(f,x+1,t) & (VALUE(f,x,t)=VALUE(f,x+1,t))]

RE

Assignment # 9.2a,b Key

1. Let sets A be recursive (decidable) and B be re non-recursive (undecidable).
   Consider C = B-A . For (a)-(c), either show sets A and B with the specified property or demonstrate that this property cannot hold.

a) Can C be recursive?

YES. Consider A = $\aleph$. B = Halt. C = {}

b) Can C be re, non-recursive?

YES. Consider A = {}. B = Halt. C = HALT

Assignment # 9.2c Key

c) Can C be non-re?

No. Can semi-decide C as follows.

First B-A = B∩~A and ~A is itself decidable and so re.

Thus, just need to show that the intersection of two re sets is also re

Let B = dom(g_B) and ~A be dom($g_{\sim A}$)

Define g_C by $g_C(x) = \exists t\ (STP(g_B,x,t)\ \&\ STP(g_{\sim A},x,t))$.

Clearly the domain of g_C is the intersection of the domains of g_B and $g_{\sim A}$ and so C = B-A is RE.