Assignment # 9.1a Key

1. Use quantification of an algorithmic predicate to estimate the complexity (decidable, re, co-re, non-re) of each of the following, (a)-(d):

(a) \{ f \mid f \text{ is a Fibonacci function, i.e. } f(0)=f(1)=1 \text{ and } f(x+2)=f(x)+f(x+1) \}

\( \forall x \exists t [\text{STP}(f,x,t) \& \text{STP}(f,x+1,t) \& \text{STP}(f,x+2,t) \& (x=0 \Rightarrow \text{VALUE}(f,x,t)=0 \& x=1 \Rightarrow \text{VALUE}(f,x,t)=1 \& x>1 \Rightarrow (\text{VALUE}(f,x+2,t) = \text{VALUE}(f,x,t)+\text{Value}(x+2,t)))] \)

Non-re, Non co-re
Assignment # 9.1b Key

b) \( f \mid |\text{range}(f)| > 1 \)}

\( \exists <x, y, t> [\text{STP}(f, x, t) \& \text{STP}(f, y, t) \& (\text{VALUE}(f, x, t) \neq (\text{VALUE}(f, y, t)))] \)

RE
Assignment # 9.1c Key

c) \{ <f,x> \mid \text{if } f(x) \text{ converges, it does so in more than } (2^x) \text{ units of time} \}

\sim \text{STP}(f, x, 2^x)
REC / DEC
Assignment # 9.1d Key

d) \{ f \mid f(x) = f(x+1) \text{ for at least one value of } x \} \]

\[ \exists <x,t> \ [ \text{STP}(f,x,t) \land \text{STP}(f,x+1,t) \land (\text{VALUE}(f,x,t) = \text{VALUE}(f,x+1,t))] \]

RE
1. Let sets $A$ be recursive (decidable) and $B$ be re non-recursive (undecidable).
   Consider $C = B - A$. For (a)-(c), either show sets $A$ and $B$ with the specified property or demonstrate that this property cannot hold.
   a) Can $C$ be recursive?
      YES. Consider $A = \emptyset$. $B = Halt$. $C = \{\}$
   b) Can $C$ be re, non-recursive?
      YES. Consider $A = \{\}$. $B = Halt$. $C = HALT$
c) Can $C$ be non-re?

No. Can semi-decide $C$ as follows.

First $B-A = B \cap \neg A$ and $\neg A$ is itself decidable and so re.

Thus, just need to show that the intersection of two re sets is also re

Let $B = \text{dom}(g_B)$ and $\neg A$ be $\text{dom}(g_{\neg A})$

Define $g_C$ by $g_C(x) = \exists t \ (\text{STP}(g_B, x, t) \ & \ & \text{STP}(g_{\neg A}, x, t))$.

Clearly the domain of $g_C$ is the intersection of the domains of $g_B$ and $g_{\neg A}$ and so $C = B-A$ is RE.