Assignment # 5.1 (M-N)

a. \( L = \{ x#y \mid x, y \in \{0,1\}^+ \text{ and } y \text{ is the twos complement of } x \} \)

Let \( R_L \) be the right invariant equivalence class defined by M-N for \( L \). Consider the pair of equivalent classes \([010^i#] \) and \([010^j#]\), \( i \neq j \), \( i, j > 0 \). \( 010^i#110^i \in L \) but \( 010^j#110^i \notin L \). Thus, for each distinct pair, \( i, j \), \( i \neq j \), \([010^i] \neq [010^j] \) and hence \( R_L \) has infinite index.

\( L \) is not Regular by Myhill-Nerode Theorem.
Assignment # 5.1 (PL)

a. \( L = \{ x\#y \mid x, y \in \{0,1\}^* \text{ and } y \text{ is the twos complement of } x \} \)

PL: Gives me \( N > 0 \) associated with \( L \)
Me: Choose \( w = 10^N\#110^N \) which is in \( L \)
PL: States \( w = xyz, |xy| \leq N, |y| > 0, xy^i z \in L \) for all \( i \geq 0 \)
Me: Choose \( i = 0 \).
Case 1: If the initial 1 is erased, we have \( 0^{N-|y|-1}\#110^N \) which is not in \( L \).
Case 2: If only 0’s are affected, we have \( 10^{N-|y|}\#110^N \), which is not in \( L \) when \( |y| > 0 \) (would need to end with \#110^{N-|y|} \).

Thus, \( L \) is not Regular by Pumping Lemma for Regular Languages.
Assignment # 5.1b (M-N)

b. $L = \{ a^i b^j c^k \mid i > k \text{ or } j > k \text{ or } k > i \}$

Let $R_L$ be the right invariant equivalence relation defined for $L$ by M-H. Consider $[a^i b^i]$ and $[a^j b^j]$ for $i \neq j$. $a^i b^j c^i \notin L$ but $a^j b^i c^j \in L$

Thus, for any two distinct $i, j$, $i \neq j$, $[a^i b^i] \neq [a^j b^j]$

This is a bit different than all our other cases in that we focus on a pattern that leads to a string not in $L$ versus one in $L$ – that’s a bit of a flip.
b. \( L = \{ a^i b^j c^k \mid i > k \text{ or } j > k \text{ or } k > i \} \). I found it easy to do the complement of \( L \). As regular are closed under complement, if the complement of \( L \) is not Regular, then neither is \( L \). \( L^C = \{ a^i b^j c^k \mid i \geq 0 \} \cup b^+ (a+c)^+ + c^+ (a+b)^+ \). Note that the latter two parts are regular and do not overlap the first part.

Me: \( L^C \) is regular

PL: Gives me \( N > 0 \) associated with \( L^C \)

Me: Choose \( w = a^N b^N c^N \) which is in \( L^C \)

PL: States \( w = xyz, |xy| \leq N, |y| > 0, xy^i z \in L^C \) for all \( i \geq 0 \)

Me: Choose \( i = 0 \). This says that \( xz = a^{N-|y|} b^N c^N \in L \). But, since \( |y| > 0 \) and there are no order problems (b’s before a’s or c’s, or c’s before a’s or b’s), this string is not in \( L^C \).

Thus, \( L^C \) is not Regular by Pumping Lemma for Regular Languages and, by closure of Regular under complement, \( L \) is not Regular.
c. \( L = \{ xw x | x, w \in \{a,b\}^+ \text{ and } |x| = |w| \} \)

I attack this with M-N. Let \( R_L \) be the right invariant equivalence relation defined for \( L \) by M-H. Consider \([a^ib^{i+1}]\) and \([a^ib^{j+1}]\) \( i < j \).

\( a^ib^{i+1}a \in L \) but \( a^ib^{i+1}a \notin L \), when \( j < i \), as last part ending \( b \) means that first part must also end in \( b \) and so each part ending in \( b \) must be preceded by \( i \) \( a \)'s. The problem then is that the middle part (the \( w \) part) would have length \( j+1 \) which is longer than the start and end parts (the \( x \) parts) which are of length \( i+1 \).

Thus, for any two distinct \( i, j, i \neq j, [a^ib^{i+1}] \neq [a^ib^{j+1}] \). Note that, if \( i < j \), we have from above; if \( i > j \), then reverse roles of \( i \) and \( j \) and get result from above.

\( L \) is not Regular by Myhill-Nerode Theorem.
Assignment # 5.1c (PL)

c. \( L = \{ x \; w \; x \mid x, \; w \in \{a,b\}^+ \text{ and } |x| = |w| \} \)

PL: Gives me \( N>0 \) associated with \( L \)
Me: Choose \( v=a^Nba^Nba^Nb \) which is in \( L \)
PL: States \( v=xyz, \; |xy|\leq N, \; |y|>0, \; xy^iz \in L \text{ for all } i\geq0 \)
Me: Choose \( i = 0 \). \( a^{N-|y|}ba^Nba^Nb \notin L \) since third \( b \) must have same number of \( a \)'s preceding it as first \( b \), so middle part is \( a^{Nba^{|y|}} \), which is longer that the starting part.

Thus, \( L \) is not Regular by Pumping Lemma for Regular Languages.
2. Write a regular (right linear) grammar that generates \( L = \{ w \mid w \in \{0,1\}^* \text{ and } w \text{ interpreted as a binary number has a remainder of 3 or 4 when divided by 6 } \} \).

\[
\begin{align*}
\langle 0 \rangle & \rightarrow 0 \langle 0 \rangle \mid 1 \langle 1 \rangle \\
\langle 1 \rangle & \rightarrow 0 \langle 2 \rangle \mid 1 \langle 3 \rangle \\
\langle 2 \rangle & \rightarrow 0 \langle 4 \rangle \mid 1 \langle 5 \rangle \\
\langle 3 \rangle & \rightarrow 0 \langle 0 \rangle \mid 1 \langle 1 \rangle \mid \lambda \\
\langle 4 \rangle & \rightarrow 0 \langle 2 \rangle \mid 1 \langle 3 \rangle \mid \lambda \\
\langle 5 \rangle & \rightarrow 0 \langle 4 \rangle \mid 1 \langle 5 \rangle
\end{align*}
\]
2. Present a Mealy Model finite state machine that reads an input $x \in \{0, 1\}^*$ and produces the binary number that represents the result of adding binary 101 to $x$ (assumes all numbers are positive, including results). Assume that $x$ is read starting with its least significant digit. Examples: 0010 $\rightarrow$ 0111; 0101 $\rightarrow$ 1010; 0001 $\rightarrow$ 0110; 0111 $\rightarrow$ 1100