## Assignment 2 Key

## Question 1)

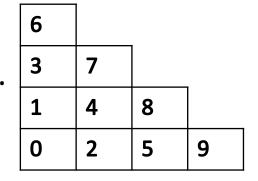
Consider the following function, **p**, from **x** to **x**. p(0) = 0; p(x+1) = p(x) + x + 2Prove inductively that  $p(x) = (x^2 + 3x)/2$ Base: x = 0;  $(x^2 + 3x)/2 = (0 + 0)/2 = 0 = p(0)$  by definition. IH: Assume  $p(x) = (x^2 + 3x)/2$  for some  $x = k \ge 0$ IS: Show for  $\mathbf{x} = \mathbf{k} + \mathbf{1}$ .  $((k+1)^{2} + 3(k+1))/2 = (k^{2} + 2k + 1 + 3k + 3)/2 = (k^{2} + 3k + 2k + 4)/2 =$  $(k^{2} + 3k)/2 + k + 2 = p(k) + k + 2$  by IH. But p(k+1) = p(k) + k + 2 by definition, proving what was desired.

## Question 2)

Consider the following function, **q**, from **x** to **x**. q(0) = 0; q(y+1) = q(y) + y + 1Prove inductively that  $q(y) = (y^2 + y)/2$ Base: y = 0;  $(y^2 + y)/2 = (0 + 0)/2 = 0 = q(0)$  by definition. IH: Assume  $q(x) = (y^2 + y)/2$  for some  $y = k \ge 0$ IS: Show for **y** = **k+1**.  $((k+1)^{2} + k+1)/2 = (k^{2} + 2k + 1 + k + 1)/2 = (k^{2} + k + 2k + 2)/2 =$  $(k^{2} + k)/2 + k + 1 = q(k) + k + 1$  by IH. But q(k+1) = q(k) + k + 1 by definition, proving what was desired.

## Question 3)

Consider the two variable function, **t**, from  $x \times x$  to x  $t(x,y) = (x^2 + 3x + 2xy + y + y^2)/2$ This embodies **p** and **q**, in that p(x) = t(x,0) and q(y) = t(0,y). Fill in the following matrix with values of t(x,y)along the first four left to right diagonals (x is horizontal axis; y is vertical axis).



Explain what the pattern is and how you could continue to fill in diagonals without ever looking back at the formula.

Every diagonal starts at the top y cell (x=0) in new diagonal as one more than the value of the last value written (y=0) in previous diagonal and increment values by one for each subsequent cell on the diagonal. For our case, next diagonal's values are 10, 11, 12, 13, 14.