Assignment 2 Key

Question 1)

Consider the following function, **p**, from **א** to **א**.
$p(0) = 0$; $p(x+1) = p(x) + x + 2$
Prove inductively that **$p(x) = (x^2 + 3x)/2$**

Base: **$x = 0$; $(x^2 + 3x)/2 = (0 + 0)/2 = 0 = p(0)$** by definition.

IH: Assume **$p(x) = (x^2 + 3x)/2$ for some $x = k \geq 0$**

IS: Show for **$x = k+1$**.

$((k+1)^2 + 3(k+1))/2 = (k^2 + 2k + 1 + 3k +3)/2 = (k^2 + 3k + 2k + 4)/2 =$

$(k^2 + 3k)/2 + k + 2 = p(k) + k + 2$ by IH.

But **$p(k+1) = p(k) + k + 2$** by definition, proving what was desired.

Question 2)

Consider the following function, **q**, from **א** to **א**.
$q(0) = 0$; $q(y+1) = q(y) + y + 1$
Prove inductively that **$q(y) = (y^2 + y)/2$**

Base: **$y = 0$; $(y^2 + y)/2 = (0 + 0)/2 = 0 = q(0)$** by definition.

IH: Assume **$q(x) = (y^2 + y)/2$ for some $y = k \geq 0$**

IS: Show for **$y = k+1$**.

$((k+1)^2 + k+1)/2 = (k^2 + 2k + 1 + k +1)/2 = (k^2 + k + 2k + 2)/2 =$

$(k^2 + k)/2 + k + 1 = q(k) + k + 1$ by IH.

But **$q(k+1) = q(k) + k + 1$** by definition, proving what was desired.

Question 3)

Consider the two variable function, **t**, from $\aleph \times \aleph$ to $\aleph$
$\mathbf{t(x,y) = (x^2 + 3x + 2xy + y + y^2)/2}$
This embodies **p** and **q**, in that **p(x) = t(x,0)** and **q(y) = t(0,y)**.
Fill in the following matrix with values of **t(x,y)**
along the first four left to right diagonals
(x is horizontal axis; y is vertical axis).

6			
3	7		
1	4	8	
0	2	5	9

Explain what the pattern is and how you could continue to fill in diagonals without ever looking back at the formula.

Every diagonal starts at the top y cell (x=0) in new diagonal as one more than the value of the last value written (y=0) in previous diagonal and increment values by one for each subsequent cell on the diagonal. For our case, next diagonal's values are 10, 11, 12, 13, 14.