Assignment 1 Key
Question 2)

Prove the following: Let $R$ be an equivalence relation over some universe $U$, and let $C_a$ be the class of all elements in $U$ equivalent to the element $a$, i.e., $C_a = \{x | x \in U \land a R x \}$, and $C_b$ be the class of all elements in $U$ equivalent to the element $b$, i.e., $C_b = \{x | x \in U \land b R x \}$.

Prove that either $C_a = C_b$ or $C_a \cap C_b = \emptyset$. 
Proof: First, given any two subsets of any universe $U$, either the subsets overlap or they have no common elements. Thus, for $C_a$ and $C_b$, either $C_a \cap C_b = \Phi$, and we are done, or $C_a \cap C_b \neq \Phi$. In this latter case, there is at least one element $z \in U$, such that $z \in C_a$ and $z \in C_b$. Thus, $a \ R \ z$ and $b \ R \ z$. Since $R$ is an equivalence relation, $b \ R \ z$ implies $z \ R \ b$ by symmetry and then $a \ R \ b$ by transitivity and $b \ R \ a$ by symmetry. Thus, $b \in C_a$ and $a \in C_b$, and consequently, based on symmetry, all members of $C_a$ are in $C_b$, and vice versa. Mutual inclusion then means $C_a = C_b$, completing the proof. Note that reflexivity guarantees that all elements of $U$ are in some class, so $R$ partitions the universe $U$. 