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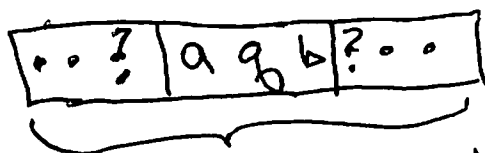
SEMI-THE

$\alpha \rightarrow \beta$

AND TURING MACHINES
(SHORT STRING REPLACEMENT
IN IDS)

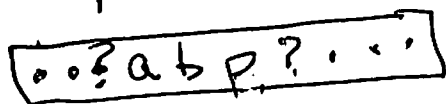
CHANGES TO TM INSTANTANEOUS DESCRIPTION
AS TM MAKES MOVES

ID IS SOME FINITE DESCRIPTION. USE
ONE WITH LEFT/RIGHT SIDE STRINGS AND STATE

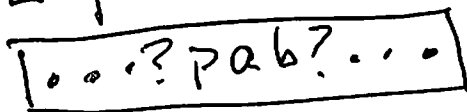


INCLUDES ALL NON-BLANK SQUARES

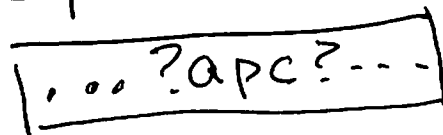
$q \rightarrow R$



$q \rightarrow L$



$q \rightarrow C$



TYPICALLY USE SOME LEFT/RIGHT END MARKERS

E.G.

R	...	...	R
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SO CAN EXPAND AND CONTRACT
FOR EXAMPLE $aqbR \Rightarrow abpOR$ IF RIGHT MOVE
OR $Rqb \Rightarrow Rqob$ IF LEFT MOVE

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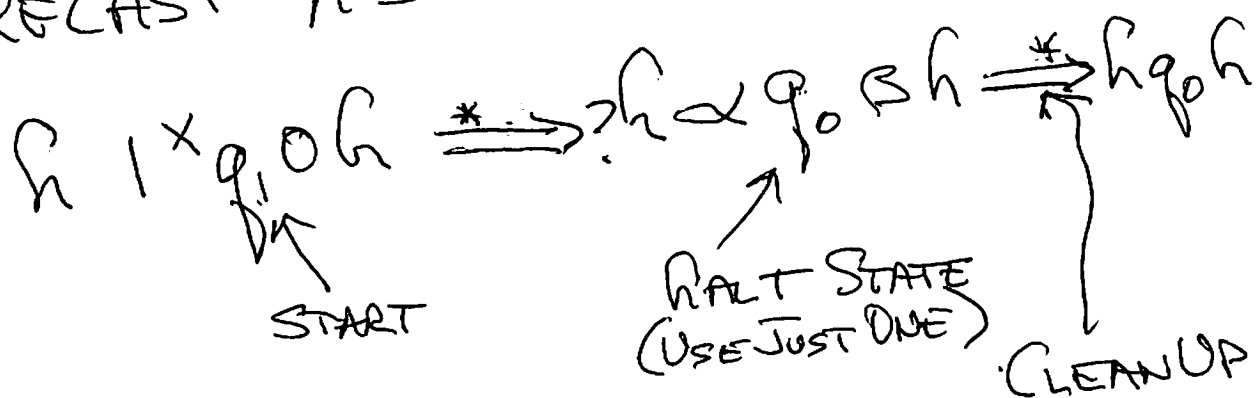
MORE DETAILS ON ST SIMULATING TM

$h \alpha q_0 \beta h$

IN STATE q , READING q

SEE PAGE 360 FOR DETAILED SIM

TM HALT PROBLEM CAN BE
RECAST AS ST WORD PROBLEM



THUS, IF TM, M , ACCEPT x ,
THAT IS, M HALTS WHEN STARTED ON x ,
THEN ST_M (ST SIMULATING M) REWRITES

$h \mid x q_0 \beta h$ AS $h q_0 h$ } $h \mid x q_0 \beta h \Rightarrow^* h q_0 h$

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UNSOLVABILITY OF ST WORD PROBLEM

PRIOR SHOWS A SOLUTION TO ST WORD PROBLEM:
GIVEN ST, S , AND WORDS w_1, w_2 ,
IS IT TRUE THAT $w_1 \xrightarrow[S]{*} w_2$

IS UNDECIDABLE

UNSOLVABILITY OF PCP

CAN MIMIC ST DERIVATION BY SOLUTION
TO INSTANCE OF PCP (POSTCORRESPONDENCE PROB)

EXAMPLE: $aba \rightarrow ab; a \rightarrow aa; b \rightarrow a$ (ST, S)

QUESTION: $bbbb \xrightarrow[S]{*} aa$

PCP INSTANCE

$\xrightarrow{X}$ $\xrightarrow{Y}$ $[b b b b *, \underline{a} b, \underline{a} a, \underline{a} a, \underline{a}, \underline{a}, \square]$
 $[\quad, \underline{a} b a, \underline{a} b a, \underline{a}, \underline{a}, \underline{b}, \underline{b}, \underline{* a a}]$
MUST START RULES MUST END

ALSO $*, \underline{*}, \underline{a}, \underline{a}, \underline{b}, \underline{b}$
 $\underline{*}, \underline{*}, \underline{a}, \underline{a}, \underline{b}, \underline{b}$

USED FOR PART NOT REWRITTEN

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ST SOLUTION MAPPED TO PCP

MUST START [b b b b *

AS ONLY PAIR THAT STARTS WITH SAME CHARACTER

[b b b b *

[MIMIC THREE STEPS OF $b \rightarrow a$ TO 1, 2, 4 CHARACTERS

[b b b b * a b a a *

[b b b b *

MIMIC ONE STEP OF $aba \rightarrow ab$

[b b b b * a b a a * a b a *

[b b b b * a b a a *

MIMIC ONE STEP OF $aba \rightarrow ab$

[b b b b * a b a a * a b a * a b *

[b b b b * a b a a * a b a *

MIMIC ONE STEP OF $b \rightarrow a$

[b b b b * a b a a * a b a * a b * a a

[b b b b * a b a a * a b a * a b :

[b b b b * a b a a * a b a * a b * a a]

[b b b b * a b a a * a b a * a b * a a]

[b b b b * a b a a * a b a * a b * a a]

LAST STEP

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PCP CONSEQUENCES

$P: \vec{X} = (x_1, \dots, x_n) \quad \vec{Y} = (y_1, \dots, y_n)$

① AMBIGUITY OF CFG IS UNDECIDABLE

$S \rightarrow A | B$
 $A \rightarrow x_i A [i] | x_i [i] \quad 1 \leq i \leq n$
 $B \rightarrow y_i B [i] | y_i [i] \quad 1 \leq i \leq n$
 $k > 0$

P HAS SOLN. IFF

$A \xRightarrow{*} x_{i_1} \dots x_{i_k} [i_k] \dots [i_1]$
 $B \xRightarrow{*} y_{i_1} \dots y_{i_k} [i_k] \dots [i_1]$

$\exists i_1, \dots, i_k \Rightarrow x_{i_1} \dots x_{i_k} \equiv y_{i_1} \dots y_{i_k}$

IFF ABOVE GRAMMAR IS AMBIGUOUS

② CONSIDER A-RULES AND B-RULES ABOVE

$L_A \cap L_B \neq \emptyset$ IFF P HAS SOLN

SO NON-EMPTYNESS (EMPTYNESS) OF INTERSECTION OF CFLS IS UNDEC.

NOTE! INTERSECTION OF CFLS IS A CSL SO EMPTYNESS OF CSL IS UNDEC.

③ CAN DO CSL EMPTYNESS. AS DIRECT PROOF

$S \rightarrow x_i S y_i^R | x_i^T y_i^R \quad 1 \leq i \leq n$

$q T a \rightarrow * T *$

$* a \rightarrow a *$

$q * \rightarrow * q$

$\bar{T} \rightarrow *$

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TRACES

A COMPUTATIONAL TRACE IS A WORD OF FORM

$\#X_0\#X_1\#\dots\#X_k\#$

WHERE $X_i \Rightarrow X_{i+1}$ $0 \leq i < k$, EACH X_i AN ID,
AND X_k IS A TERMINAL ID

WE CAN SHOW CFGS THAT ALMOST
GET TRACES (OFTEN ALTERNATING BETWEEN
IDS AND THE REVERSAL OF IDS). WHAT
THEY GET IS EVERY OTHER PAIR RIGHT

$\#X_0\#X_1\#\dots\#X_k\#$

WHERE $X_{2i} \Rightarrow X_{2i+1}$

BUT $X_{2i+1} \Rightarrow ? X_{2i+2}$

SO EVEN/ODD RIGHT, BUT ODD/EVEN
ARE NOT NEC. RIGHT

CAN GET SEPARATE GRAMMAR FOR ODD/EVEN

CAN USE $\cap$ OR QUOTIENT TO GET
ALIGNMENT

WITHOUT PROOF (SEE 3.7.6), FOR CFL L_1, L_2
 L_1/L_2 CAN BE ANY RE SET.

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COMPLEMENT OF TRACE

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WHILE TRACE REQUIRES ALL PAIRS TO BE RIGHT, COMPLEMENT REQUIRES JUST ONE ERROR.

CAN SHOW COMPLEMENTS OF TRACES ARE CFLs. MOREOVER, IF MACHINE FOR WHICH WE HAVE TRACES ACCEPTS $\emptyset$ THEN THERE ARE NO TERMINATING TRACES AND COMPLEMENT IS Σ^* .

THIS IS WHY WE CANNOT DECIDE IF A CFG, G , IS SUCH THAT $L(G) = \Sigma^*$. WE CAN DECIDE IF $L(G) = \emptyset$, $|L(G)| = \text{INF}$.

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$$\mathcal{L}(G) = \mathcal{L}(G)^2$$

LET G BE AN ARBITRARY CFG (OR CSG)

LET $L = \mathcal{L}(G)$, WANT TO SHOW $L = L^2$ IS UNDEC.

$$\text{NOTE! } L^2 = \{xy \mid x, y \in L\}$$

FIRST, RECALL THAT $L = \Sigma^*$ IS UNDEC.
FOR CFG (AND CSG)

CLAIM: $L = \Sigma^*$ IFF

(1) $\Sigma \cup \{\lambda\} \subseteq L$; AND (NOTE: DECIDABLE)

(2) $L \cdot L = L$

CLEARLY IF $L = \Sigma^*$ THEN (1) AND (2) HOLD

CONVERSELY, $\Sigma^* \subseteq L^* = \bigcup_{n \geq 0} L^n \subseteq L$

↑
FOLLOWS
FROM (1)

↑
FOLLOWS
FROM (2)
SINCE IF
 $L = L^2$ THEN
 $L = L^3$, ETC.

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COMPLEXITY (NP)

$P = \{ P \mid P \text{ is a decision problem solvable in polynomial time in terms of representations of instances of } P \text{ using a det. TM} \}$

P CONTAINS DEPTH FIRST SEARCH SOLVABLE PROBLEMS - E.G., GARBAGE COLLECTION, GRAPH CONNECTIVITY, RELATIVE PRIMALITY, MEMBERSHIP IN CFL (CKY), LINEAR PROG. OVER REALS (NOT CONSTRAINED TO INTEGER), PRIMALITY

$NP = \{ P \mid P \text{ is a decision problem solvable in polynomial time ... using a non-det. TM} \}$

ALSO, NP IS

$NP = \{ P \mid \text{A SOLUTION TO AN INSTANCE OF } P \text{ CAN BE VERIFIED IN DET. POLY TIME} \}$

CAN ALSO USE NOTION OF UNBOUNDED PARALLELISM

A PROBLEM IN NP IS BOOLEAN SATISFIABILITY CAN SOLVE WITH TRUTH TABLE BY GUESSING A SOLUTION (OR CHECK A PROPOSED ASSIGNMENT OF VALUES IN LINEAR TIME)

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COMPLEXITY OTHER CLASSES

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CO-NP: COMPLEMENT OF NP

NP-HARD: SUPERSET OF NP, WHEN
 $P \in NP, Q \in NP\text{-HARD} \Rightarrow P \leq_P Q$

NP-COMPLETE: NP-HARD AND IN NP

QSAT IS NP-HARD BUT MAY NOT
BE IN NP

PSPACE: SOLVABLE IN POLYNOMIAL SPACE

QSAT $\in$ PSPACE

EXP: PROBLEMS SOLVABLE IN EXPONENTIAL TIME

BIG QUESTION

$$P = NP?$$

IF $P = NP$ THEN ALL NP PROBLEMS ARE IN P

IF $P \neq NP$ THEN ALL NP-C PROBLEMS ARE IN EXP
BUT NOT IN P