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SEMI-THUE AND TURING MACHINES

$\alpha \rightarrow \beta$

(SHORT STRING REPLACEMENT IN IDs)

CHANGES TO TM INSTANTANEOUS DESCRIPTION AS TM MAKES MOVES

ID IS SOME FINITE DESCRIPTION. USE ONE WITH LEFT/RIGHT SIDE STRINGS AND STATE

$\underbrace{| \cdot \cdot ? \,|\, a\, q\, b \,|\, ? \cdot \cdot |}_{\text{INCLUDES ALL NON-BLANK SQUARES}}$

$q\,b\,R\,p$

$| \cdot \cdot ?\, a\, b\, p\, ? \cdot \cdot \cdot |$

$q\,b\,L\,p$

$| \cdot \cdot \cdot ?\, p\, a\, b\, ? \cdot \cdot \cdot |$

$q\,b\,c\,p$

$| \cdot \cdot \cdot ?\, a\, p\, c\, ? \cdot \cdot \cdot |$

TYPICALLY USE SOME LEFT/RIGHT END MARKERS

E.G. $| h \cdot \cdot \cdot \cdot \quad \cdot \cdot \cdot \cdot h |$

SO CAN EXPAND AND CONTRACT

FOR EXAMPLE $a q b h \Rightarrow a b p 0 h$ IF RIGHT MOVE

OR $h q^0 b \Rightarrow h q 0 b$ IF LEFT MOVE

MORE DETAILS ON ST SIMULATING TM

$h \alpha q a \beta h$

IN STATE q, READING a

SEE PAGE 360 FOR DETAILED SIM

TM HALT PROBLEM CAN BE RECAST AS ST WORD PROBLEM

$$h 1^x q_1 0 h \xRightarrow{*} h \alpha q_0 \beta h \xRightarrow{*} h q_0 h$$

START

HALT STATE (USE JUST ONE)

CLEANUP

THUS, IF TM, M, ACCEPT x, THAT IS, M HALTS WHEN STARTED ON x, THEN ST_M (ST SIMULATING M) REWRITES $h1^xq_10h$ AS hq_0h

$$h1^xq_10h \xRightarrow{*} hq_0h$$

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UNSOLVABILITY OF ST WORD PROBLEM

PRIOR SHOWS A SOLUTION TO ST WORD PROBLEM: GIVEN ST, S, AND WORDS w_1, w_2, IS IT TRUE THAT $w_1 \xRightarrow[S]{*} w_2$

IS UNDECIDABLE

UNSOLVABILITY OF PCP

CAN MIMIC ST DERIVATION BY SOLUTION TO INSTANCE OF PCP (POST CORRESPONDENCE PROB)

EXAMPLE: $aba \to ab$; $a \to aa$; $b \to a$ (ST, S)

QUESTION: $bbbb \xRightarrow[S]{*} aa$

PCP INSTANCE

$$\begin{array}{lllllllll}
\vec{x} & [\,bbbb*, & ab, & \underline{ab}, & aa, & \underline{aa}, & a, & \underline{a}, &] \\
\vec{y} & [\,, & \underline{aba}, & aba, & \underline{a}, & a, & \underline{b}, & b, & \underline{*}aa]
\end{array}$$

MUST START

RULES

MUST END

ALSO

$$\begin{array}{llllll}
, & \underline{}, & a, & \underline{a}, & b, & \underline{b} \\
\underline{*}, & *, & \underline{a}, & a, & \underline{b}, & b
\end{array}$$

USED FOR PART NOT REWRITTEN

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ST SOLUTION MAPPED TO PCP

MUST START [bbbb*
[

AS ONLY PAIR THAT STARTS WITH SAME CHARACTER

[bbbb*
[

MIMIC ~~THREE~~STEPS OF b→a TO 1,2,4 CHARACTERS

[bbbb*abaa*
[bbbb*

MIMIC ONE STEP OF aba→ab

[bbbb*abaa*aba*.
[bbbb*abaa*
[bbbb*abaa*

MIMIC ONE STEP OF aba→ab

[bbbb*abaa*aba*ab*
[bbbb*abaa*aba*
[bbbb*abaa*

MIMIC ONE STEP OF b→a

[bbbb*abaa*aba*ab*aa
[bbbb*abaa*aba*ab

[bbbb*abaa*aba*ab*aa]
[bbbb*abaa*aba*ab*aa]
[bbbb*abaa*

LAST STEP

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PCP CONSEQUENCES

P: $\vec{X} = (x_1, \ldots, x_n)$ $\vec{Y} = (y_1, \ldots, y_n)$

① AMBIGUITY OF CFG IS UNDECIDABLE

$S \rightarrow A \mid B$

$A \rightarrow x_i A [i] \mid x_i [i]$ $1 \le i \le n$

$B \rightarrow y_i B [i] \mid y_i [i]$ $1 \le i \le n$

$k > 0$

P HAS SOLN. IFF $\exists i_1 \ldots i_k \ni$

$A \overset{*}{\Rightarrow} x_{i_1} \cdots x_{i_k} [i_k] \cdots [i_1]$

$B \overset{*}{\Rightarrow} y_{i_1} \cdots y_{i_k} [i_k] \cdots [i_1]$

$x_{i_1} \cdots x_{i_k} \equiv y_{i_1} \cdots y_{i_k}$

IFF ABOVE GRAMMAR IS AMBIGUOUS

② CONSIDER A-RULES AND B-RULES ABOVE

$L_A \cap L_B \neq \phi$ IFF P HAS SOLN

SO

NON-EMPTINESS (EMPTINESS) OF INTERSECTION OF CFLs IS UNDEC.

NOTE: INTERSECTION OF CFLs IS A CSL SO EMPTINESS OF CSL IS UNDEC.

③ CAN DO CSL EMPTINESS AS DIRECT PROOF

$S \rightarrow x_i S y_i^R \mid x_i T y_i^R$ $1 \le i \le n$

$a T a \rightarrow * T *$

$* a \rightarrow a *$

$a * \rightarrow * a$

$T \rightarrow *$

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TRACES

A COMPUTATIONAL TRACE IS A WORD OF FORM

$\#X_0\#X_1\# \dots \#X_k\#$

WHERE $X_i \Rightarrow X_{i+1}$ $0 \le i < k$, EACH X_i AN ID, AND X_k IS A TERMINAL ID

WE CAN SHOW CFGs THAT ALMOST GET TRACES (OFTEN ALTERNATING BETWEEN IDs AND THE REVERSAL OF IDs), WHAT THEY GET IS EVERY OTHER PAIR RIGHT

$\#X_0\#X_1\# \dots \#X_k\#$

WHERE $X_{2i} \Rightarrow X_{2i+1}$

BUT $X_{2i+1} \stackrel{?}{\Rightarrow} X_{2i+2}$

SO EVEN/ODD RIGHT, BUT ODD/EVEN ARE NOT NEC. RIGHT

CAN GET SEPARATE GRAMMAR FOR ODD/EVEN

CAN USE $\cap$ OR QUOTIENT TO GET ALIGNMENT

WITHOUT PROOF (SEE 376), FOR CFL L_1, L_2

L_1/L_2 CAN BE ANY RE SET.

COMPLEMENT OF TRACE

WHILE TRACE REQUIRES ALL PAIRS TO BE RIGHT, COMPLEMENT REQUIRES JUST ONE ERROR.

CAN SHOW COMPLEMENTS OF TRACES ARE CFLs. MOREOVER, IF MACHINE FOR WHICH WE HAVE TRACES ACCEPTS $\emptyset$ THEN THERE ARE NO TERMINATING TRACES AND COMPLEMENT IS Σ^*.

THIS IS WHY WE CANNOT DECIDE IF A CFG, G, IS SUCH THAT $\mathcal{L}(G) = \Sigma^*$

WE CAN DECIDE IF $\mathcal{L}(G) = \emptyset$, $|\mathcal{L}(G)| = \text{INF.}$

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$$\mathcal{L}(G) = \mathcal{L}(G)^2$$

LET G BE AN ARBITRARY CFG (OR CSG)

LET $L = \mathcal{L}(G)$. WANT TO SHOW $L = L^2$ IS UNDEC.

NOTE: $L^2 = \{xy \mid x, y \in L\}$

FIRST, RECALL ~~THAT~~ $L = \Sigma^*$ IS UNDEC.

FOR CFG (AND CSG)

CLAIM: $L = \Sigma^*$ IFF (NOTE: DECIDABLE)

(1) $\Sigma \cup \{\lambda\} \subseteq L$; AND

(2) $L \cdot L = L$

CLEARLY IF $L = \Sigma^*$ THEN (1) AND (2) HOLD

CONVERSELY, $\Sigma^* \subseteq L^* = \bigcup_{n \geq 0} L^n \subseteq L$

FOLLOWS FROM (1)

FOLLOWS FROM (2) SINCE IF $L = L^2$ THEN $L = L^3$, ETC.

COMPLEXITY (NP)

P = { p | p IS A DECISION PROBLEM SOLVABLE IN POLYNOMIAL TIME IN TERMS OF REPRESENTATIONS OF INSTANCES OF p USING A DET. TM }

P CONTAINS DEPTH FIRST SEARCH SOLVABLE PROBLEMS – E.G., GARBAGE COLLECTION, GRAPH CONNECTIVITY, RELATIVE PRIMALITY, MEMBERSHIP IN CFL (CKY), LINEAR PROGR. OVER REALS (NOT CONSTRAINED TO INTEGER), PRIMALITY

NP = { p | p IS A DECISION PROBLEM SOLVABLE IN POLYNOMIAL TIME ... USING A NON-DET. TM }

ALSO, NP IS

NP = { p | A SOLUTION TO AN INSTANCE OF p CAN BE VERIFIED IN DET. POLY TIME }

CAN ALSO USE NOTION OF UNBOUNDED PARALLELISM

A PROBLEM IN NP IS BOOLEAN SATISFIABILITY

CAN SOLVE WITH TRUTH TABLE BY GUESSING A SOLUTION (OR CHECK A PROPOSED ASSIGNMENT OF VALUES IN LINEAR TIME)

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COMPLEXITY
OTHER CLASSES

CO-NP: COMPLEMENT OF NP

NP-HARD: SUPERSET OF NP, WHEN $P \in NP, Q \in NP\text{-HARD} \Rightarrow P \leq_p Q$

NP-COMPLETE: NP-HARD AND IN NP

QSAT IS NP-HARD BUT MAY NOT BE IN NP

PSPACE: SOLVABLE IN POLYNOMIAL SPACE

QSAT $\in$ PSPACE

EXP: PROBLEMS SOLVABLE IN EXPONENTIAL TIME

BIG QUESTION

$$P = NP?$$

IF P=NP THEN ALL NP PROBLEMS ARE IN P

IF P≠NP THEN ALL NP-C PROBLEMS AR IN EXP BUT NOT IN P