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# LIMITING PDA TO PUSH/POP

PUSH: PUSH($\alpha$) IS EQUIVALENT TO

$$\delta(q, a, z) \supseteq \{(p, \alpha z)\}$$

WHERE WE JUST USE

$$\delta(q, a, z) \supseteq \{(p, \text{PUSH}(\alpha))\}$$

POP: POP IS EQUIVALENT TO

$$\delta(q, a, z) \supseteq \{(p, \lambda)\}$$

WHERE WE JUST USE

$$\delta(q, a, z) \supseteq \{(p, \text{POP})\}$$

IF WANT TO SIMULATE STANDARD OPERATION OF

$$\delta(q, a, z) \supseteq \{(p, \alpha)\}$$

CAN DO

$$\delta(q, a, z) \supseteq \{(p', \text{POP})\}$$

$$\delta(p', \lambda, X) \supseteq \{(p, \text{PUSH}(\alpha))\}$$

ANY ELEMENT OF $\Gamma$ OR $\lambda$ IF ALLOWED

$$[q_0, w, \$] \vdash^* [f, \lambda, \lambda]$$

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# PDA TO CFG

$Q = (Q, \Sigma, \Gamma, \delta, q_0, Z, F)$ $\{f\}$

USED IN TECHNIQUE 1 $Z = \$$; NOT USED IN BOOK'S APPROACH SO $\emptyset$

TECHNIQUE #1:

LIMITED TO PUSH & POP

NON-TERMINALS ARE OF FORM

$\langle p, Z, q \rangle$

GOAL IS $\langle p, Z, q \rangle \stackrel{*}{\Rightarrow} w \in \Sigma^*$

WHEN $[p, w, Z] \vdash^* [q, \lambda, \lambda]$

START SYMBOL IS

$S \to \langle q_0, \$, f \rangle$

CAN ACTUALLY DO FOR EMPTY STACK ONLY BY HAVING

$S \to \langle q_0, \$, q \rangle \quad \forall q \in Q$

RULES, OTHER THAN START, ARE

$\langle q, X, p \rangle \to a \langle s, Y, t \rangle \langle t, X, p \rangle$

WHENEVER $\delta(q, a, X) \supseteq \{(s, \mathrm{PUSH}(Y))\}$

$\langle q, X, p \rangle \to a$

WHENEVER $\delta(q, a, X) \supseteq \{(p, \mathrm{POP})\}$

GOAL: $\langle q_0, \$, f \rangle \stackrel{*}{\Rightarrow} w$, WHENEVER $w \in E(Q)$

# PDA TO CFG

$Q = (Q, \Sigma, \Gamma, \delta, q_0, \emptyset, \{f\})$

TECHNIQUE #2 : LIMITED TO PUSH/POP

NON-TERMINALS ARE OF FORM

$A_{t,u}$

GOAL IS $A_{t,u} \stackrel{*}{\Rightarrow} w \in \Sigma^*$

WHEN $[t, w, \alpha] \vdash^* [u, \lambda, \alpha]$

START IS

$A_{q_0, f}$

RULES ARE

$A_{q,q} \rightarrow \lambda$ $\quad \forall q \in Q$ REFLEXIVE

$A_{t,u} \rightarrow A_{t,v} A_{v,u}$ $\quad$ TRANSITIVE

AND

$A_{t,u} \rightarrow a A_{r,s} b$

WHEN $\delta(t, a, \lambda) \supseteq \{(r, x)\}$ PUSH

& $\delta(s, b, x) \supseteq \{(s, \lambda)\}$ POP

# GREIBACH NORMAL FORM

ALL RULES, EXCEPT PERHAPS, $S \Rightarrow \lambda$,
LIMITED TO

$$A \rightarrow a\alpha \qquad A \in V, a \in \Sigma, \alpha \in V^*$$

PROVIDES LINEAR PARSE IF
WE CAN AVOID

$$\left.\begin{array}{l}\text{SHIFT / REDUCE} \\ \& \\ \text{REDUCE/REDUCE}\end{array}\right\} \text{CONFLICTS}$$

# CFL CLOSURE

EASY: UNION

$G_A = (V_A, \Sigma, R_A, S_A)$

$G_B = (V_B, \Sigma, R_B, S_B)$

$G = (\{S\} \cup V_A \cup V_B, \Sigma, \{S \to S_A | S_B\} \cup R_A \cup R_B, S]$

MODERATE: INTERSECTION WITH REGULAR

$Q_0 = (Q_0, \Sigma, \Gamma, \delta_0, q_0, \$, F_0)$ PDA, $L_0 = \mathcal{L}(Q_0)$

$Q_1 = (Q_1, \Sigma, \delta_1, q_1, F_1)$ DFA $= L_1 = \mathcal{L}(Q_1)$

DEFINE

$Q_2 = (Q_0 \times Q_1, \Sigma, \Gamma, \delta_2, \langle q_0, q_1 \rangle, \$, F_0 \times F_1)$

$\delta_2(\langle q, s \rangle, a, X) \supseteq \{(\langle q', s' \rangle, \alpha)\}, X \in \Gamma, a \in \Sigma_e$

IFF $\delta_0(q, a, X) \supseteq \{(q', \alpha)\}$

AND $\delta_1(s, a) = s'$ IF $a \in \Sigma$

$\delta_1(s, a) = s$ IF $a = \lambda$ TREAT AS NO STATE $s' = s$

ELSE

NOW INDUCTIVELY CAN SHOW

$[\langle q_0, q_1 \rangle, w, \$] \vdash^* [\langle t, s \rangle, \lambda, B]$ IFF

$[q_0, w, \$] \vdash^* [t, \lambda, B]$ IN $Q_0$

AND $[q_1, w] \vdash^* [s, \lambda]$ IN $Q_1$

SO $w \in F(Q_2)$ IFF $t \in F_0$ & $s \in F_1$ IFF $w \in F(Q_0) \cap F(Q_1)$

# CFL CLOSURE

MODERATE: SUBSTITUTION

$G = (V, \Sigma, R, S)$ $L = \mathcal{L}(G)$

SUBSTITUTION

$f(a) = L_a$ $a \in \Sigma$ $L_a$ A CFL

$G_a = (V_a, \Sigma, R_a, S_a)$ $L_a = \mathcal{L}(G_a)$

IN R, CHANGE ALL INSTANCES OF $a \in \Sigma$ IN RHS's TO $S_a$

THUS, IF ORIGINALLY,

$S \Rightarrow a_1, \ldots, a_k$

THEN, IN NEW

$S \stackrel{*}{\Rightarrow} S_{a_1} \cdots S_{a_k}$

AND THEN

$S \stackrel{*}{\Rightarrow} w_{a_1} \cdots w_{a_k}$

WHERE $w_{a_i} \in f(a_i)$

$G' = (V \cup V_A \cup V_B, \Sigma, R', S)$

$R' = R^{\text{CHANGED}} \cup R_a \ _{a \in \Sigma}$

WHERE $R^{\text{CHANGED}}$ IS AS ABOVE

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# CFL NON-CLOSURE

## INTERSECTION

BY EXAMPLE OF CONTRADICTORY CASE

$L_1 = \{a^n b^n c^m \mid n, m > 0\}$

$L_2 = \{a^m b^n c^n \mid n, m > 0\}$

$S_1 \to S_1 c \mid T_1 c \qquad S_2 \to a S_2 \mid a T_2$

$T_1 \to a T_1 b \mid a b \qquad T_2 \to b T_2 c \mid b c$

BOTH ARE CFLs

HOWEVER,

$L_1 \cap L_2 = \{a^n b^n c^n \mid n > 0\}$

WHICH IS NOT A CFL

## COMPLEMENT

BY FACT THAT CLOSURE UNDER UNION AN COMPLEMENT IMPLIES CLOSURE UNDER INTERSECTION

$\sim(\sim A \cup \sim B) = A \cap B$

# COMPLEMENT EXAMPLE FOR CFL

$L = \{ww \mid w \in \{a,b\}^+\}$
IS NOT A CFL BUT

$\bar{L} = \{z \mid |z| \text{ IS ODD AND } z \in \{a,b\}^+\}$
$\cup \{XY \mid |X| = |Y| \text{ AND } X \neq Y\}$

FIRST PART IS REGULAR
SECOND PART IS SEEN AS

$X_1 a X_2 Y_1 b Y_2$ OR $X_1 b X_2 Y_1 a Y_2$

WHERE $|X_1| = |Y_1|$, $|X_2| = |Y_2|$

BUT $X_1, X_2, Y_1, Y_2$ VALUES ARE UNIMPORTANT

SO LOOK AT AS

$X_1 a \underbrace{Y_1 X_2}_{\text{LENGTH ONLY}} b Y_2$ AND $X_1 b \underbrace{Y_1 X_2}_{\text{LENGTH ONLY}} b Y_2$

$S \to AB \mid BA$
$A \to XAX \mid a$
$B \to XBX \mid b$
$X \to a \mid b$

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# LOTS OF CLOSURE

SUBSTITUTION & INTERSECTION W/REGULAR

$f(a) = \{a, a'\}$

$g(a) = \{a'\}$

PREFIX

$h(a) = \{a\}; h(a') = \{\lambda\}$

INFIX

$L/R = h(f(L) \cap (\Sigma^* \cdot g(R)))$

SUFFIX

$x \cdot y'$ $x \in \Sigma^*$, $y \in R$

QUOTIENT WITH REGULAR

$x \cdot y'$ $xy \in L$, $y \in R$

ETC.

BUT <u>NOT</u>

QUOTIENT WITH CFL

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# MIN AND MAX

$MAX(L_1) = \{x \mid x \in L$ AND $xy \in L$ IF $|y| > 0\}$

$MIN(L_2) = \{x \mid x \in L$ AND NO PROPER PREFIX OF $x \in L\}$

$L_1 = \{a^i b^j c^k \mid k \leq i$ OR $k \leq j\}$

$MAX(L_1) = \{a^i b^j c^k \mid k = MAX(i,j)\}$ NOT CFL

$MIN(L_1) = \{\lambda\}$ REGULAR

$L_2 = \{a^i b^j c^k \mid k > i$ OR $k > j\}$

$MAX(L_2) = \{\}$ REGULAR

$MIN(L_2) = \{a^i b^j c^k \mid k = MIN(i,j) + 1\}$ NON-CFL

BOTH $L_1$ AND $L_2$ ARE CFLs

# SOLVABLE PROBLEMS
# CFLs

$w \in L$?

RUN CKY WITH CNF, G
IF S IN FINAL CELL
$w \in L$

$L = \emptyset$

REDUCE G
IF S NON-PRODUCTIVE
$L = \emptyset$; ELSE $L \neq \emptyset$

L FINITE

REDUCE G
RUN DFS(S)
IF NO LOOPS, L FINITE;
ELSE L INFINITE