10/13/2016

CKY: $G = (\{S, A, B, C, D, E, F, G, H\}, \{a, b\}, R, S)$

R: $S \to AB \mid AC \mid AD \mid AE$
$\mid BA \mid BF \mid BG \mid BH$

$A \to a$	$C \to DS$	$F \to GS$
$B \to b$	$D \to SB$	$G \to SA$
	$E \to BS$	$H \to AS$

Does babba belong to $\mathcal{L}(G)$?

	b	a	b	b	a
1	B	A	B	B	A
2	S	S	$\emptyset$	S	
3	E, D	D	E		
4	$\emptyset$	S			
5	E, C				

$\leftarrow$ babba $\notin \mathcal{L}(G)$

10/13/2016

COCKE-KASAMI-YOUNGER
CKY

$S \to AB \mid AC \mid AD \mid AE$
$\mid BA \mid BF \mid BG \mid BH$

$A \to a$ $\quad C \to DS$ $\quad F \to GS$

$B \to b$ $\quad D \to SB$ $\quad G \to SA$

$E \to BS$ $\quad H \to AS$

---

	b	a	b	b	a	a	b	a
1	B	A	B	B	A	A	B	A
2	S	S		S		S	S	
3	E,D	D	E	G	H	H,G		
4		S	S	S				
5	E,C	H,G	E,D	F,G				
6	S	S	S					
7	E,C,D	H,F,G						
8	S							

ACCEPT

10/13/2016

PUMPING LEMMA FOR CFLs

IF L IS A CFL, THEN $\exists N > 0$, SUCH THAT IF $Z \in L$ AND $|Z| \geq N$ THEN $Z = UVWXY$, WHERE $|VWX| \leq N$, $|VX| > 0$ AND $\forall i \geq 0$ $UV^iWX^iY \in L$

NOTE THAT IF $|\Sigma| = 1$ THEN PUMPING LEMMA FOR CFL DEGENERATES TO PUMPING LEMMA FOR REGULAR

THUS, IF $L \subseteq \Sigma^*$, $|\Sigma| = 1$, AND L IS NOT REGULAR THEN L IS NOT A CFL

10/13/2012

$\{ww \mid w \in \{a,b\}^*\}$ IS NOT A CFL

1. ASSUME L IS A CFL
2. PL GIVES YOU N
3. YOU CHOOSE $a^N b^N a^N b^N$
4. PL SPLITS $a^N b^N a^N b^N$ INTO $uvwxy$
   WHERE $|vwx| \leq N, |vx| > 0$
   AND STATES $\forall i \geq 0 \; uv^iwx^iy \in L$
5. a.) IF vwx IS OVER a'S ONLY
   THEN SINCE $|vwx| \leq N$
   vwx CANNOT SPAN FROM ONE OF
   THE a^N SUBSTRINGS TO OTHER,
   AND SO FOR $i=0$, WE GET EITHER
   $a^{N-|vx|} b^N a^N b^N \in L$ OR $a^N b^N a^{N-|vx|} b^N \in L$
   AND BOTH ARE NOT SO.

   b) IF vwx IS OVER b'S ONLY, THE
   SAME ARGUMENTS APPLY

   c) IF vwx SPANS a'S AND b'S THEN
   THIS MUST BE WITHIN ONE OF
   THE $a^N b^N$ OR $b^N a^N$ SUBSTRINGS
   AS $|vwx| \leq N$. IF $i=0$ THEN WE
   REDUCE SOME a'S AND SOME b'S, BUT NOT ALL,
   WITHIN JUST ONE OF THE CONSECUTIVE
   a'S AND b'S, BUT NOT THE OTHER
   AND SO THE RESULTING STRING IS NOT
   IN L.

$\cancel{\forall}$

10/13/2016

Pushdown Automata
PDA

$$A = (Q, \Sigma, \Gamma, \delta, q_0, Z_0, F)$$

States (Finite)

Lang. Alphabet (Finite)

Stack Symbols (Finite

Start State

Optional Bottom of Stack Marker

Optional Final States

Transition Function

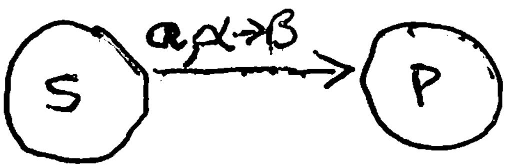
$$\delta : Q \times \Sigma_e \times \Gamma_e \to 2^{Q \times \Gamma^*}$$

Can extend to Γ^*

Can limit to Γ_e by growing states

PDA Diagrams

Some Texts Use $a, X / Y$ which is fine also



$$\delta(s, a, \alpha) \supseteq \{(p, \beta)\}$$

10/13/2016

PDA LANGUAGES
$\equiv$ CFLs

INSTANTANEOUS DESCRIPTIONS (ID)

$[q, w, \gamma]$

q - CURRENT STATE
w - REMAINING INPUT
γ - STACK CONTENTS
READ LEFT (TOP) TO RIGHT (BOTTOM)

SINGLE STEP

$[q, ax, Z\alpha] \vdash [p, x, \beta\alpha]$ IF $\delta(q, a, Z) \in (p, \beta)$

MULTISTEP $\vdash^*$ REFLEXIVE TRANSITIVE CLOSURE OF $\vdash$

GIVEN $A = (Q, \Sigma, \Gamma, \delta, q_0, Z_0, F)$
THERE CAN BE THREE NOTIONS OF ACCEPTANCE

FINAL STATE

$\mathcal{L}(A) = \{w \mid [q_0, w, Z_0] \vdash^* [f, \lambda, \beta]\}, f \in F$

EMPTY STACK

$N(A) = \{w \mid [q_0, w, Z_0] \vdash^* [q, \lambda, \lambda]\}, q \in Q$

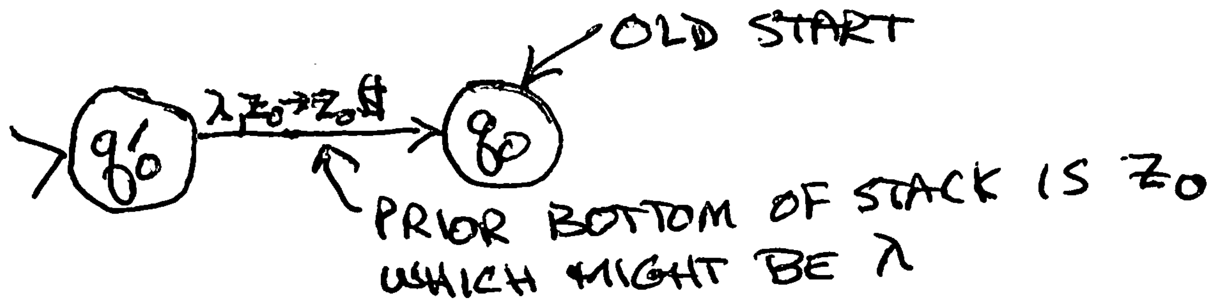
EMPTY STACK AND FINAL STATE

$E(A) = \{w \mid [q_0, w, Z_0] \vdash^* [f, \lambda, \lambda]\}, f \in F$

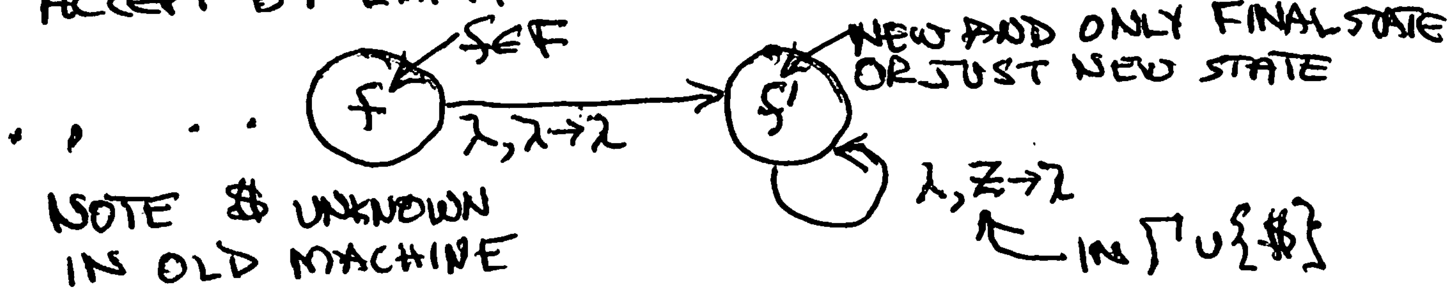
10/13/2016

EQUIVALENCY OF LANGUAGE CLASSES, $\mathcal{L}(A)$, $N(A)$, $E(A)$, WHERE A RANGES OVER ALL PDAs

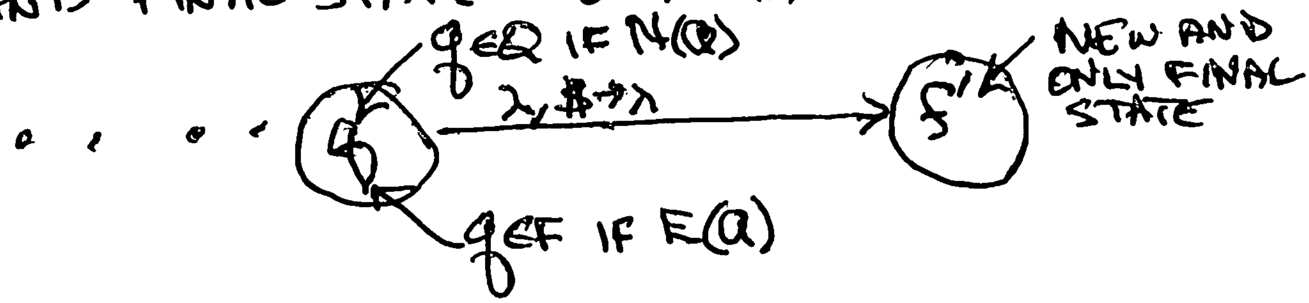
- CONVERTING ONE FORM TO ANOTHER
  FOR EACH CASE, ASSUME q_0' IS A NEW STATE (NEW START) AND \$ IS A NEW STACK SYMBOL PLACED ON BOTTOM OF STACK. FOR ALL CASES, START WITH



- CHANGE ACCEPT BY FINAL STATE TO ACCEPT BY EMPTY STACK OR EMPTY AND FINAL STATE



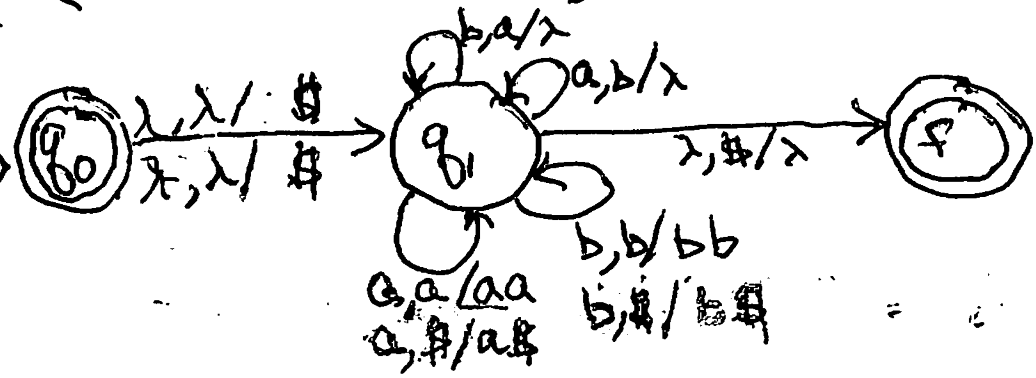
- CHANGE ACCEPT BY EMPTY STACK OR EMPTY AND FINAL STATE TO ACCEPT BY FINAL STATE



EXAMPLE PDA

$L_1 = \{ w \mid |w|_a = |w|_b \}$. ASSUME λ ON STACK

Q_1:



I USED / WHERE BOOK USES $\rightarrow$

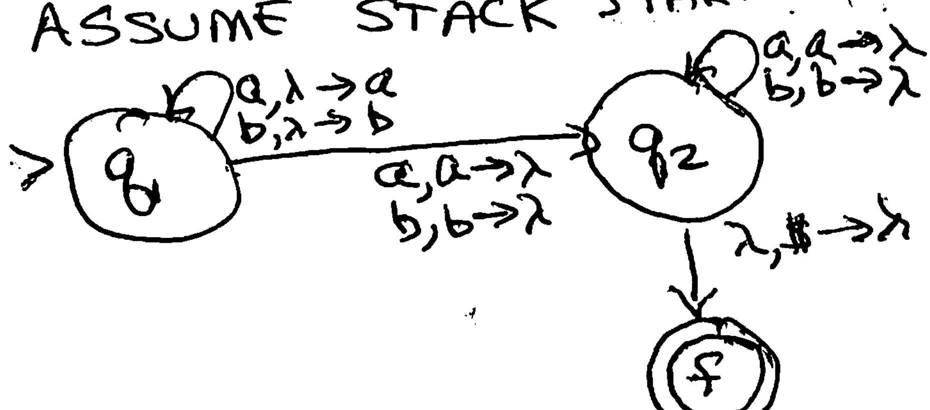
THIS GETS $L_1 = E(Q_1)$

THIS IS NON-DETERMINISTIC

$L_2 = \{ w w^R \mid w \in \{a, b\}^+ \}$

ASSUME STACK STARTS WITH \$

Q_2:

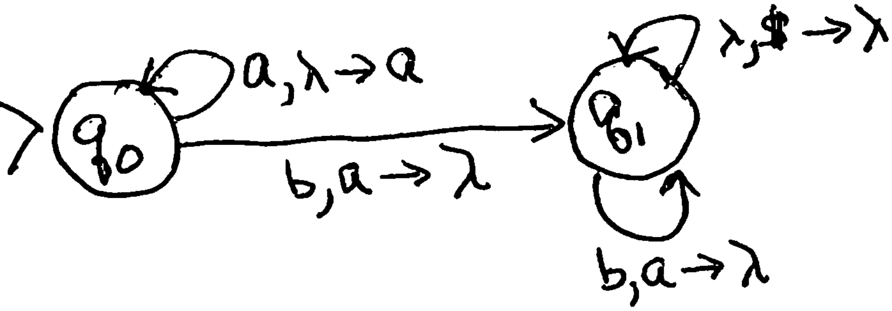


THIS GETS $L_2 = \mathcal{L}(Q_2) = E(Q_2)$

THIS IS ALSO NON-DETERMINISTIC

$L_3 = \{a^n b^n \mid n > 0\}$. ASSUME \$ ON STACK

Q_3 :



THIS GETS $L_3 = N(Q_3)$

THIS IS DETERMINSTIC

DETERMINISM FOR PDAs

1) FOR EACH $q \in Q$ & $z \in \Gamma$ & $a \in \Sigma$
   IF $|\delta(q, \lambda, z)| > 0$ THEN $|\delta(q, a, z)| = 0$

2) FOR NO $q \in Q$, $z \in \Gamma$ & $a \in \Sigma_\epsilon$
   IS $|\delta(q, a, z)| > 1$

10/13/2016

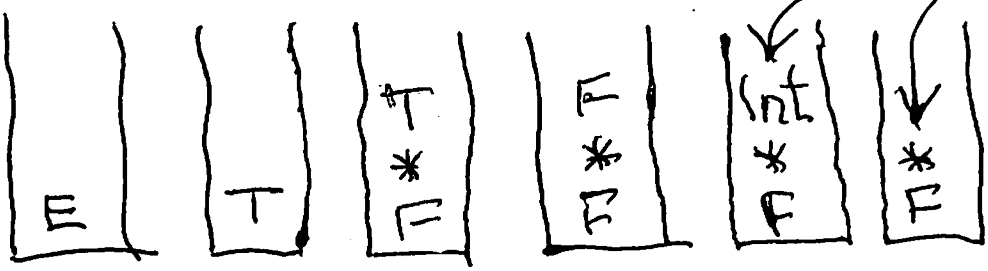
TOP-DOWN PARSER

$E \rightarrow E + T \mid T$

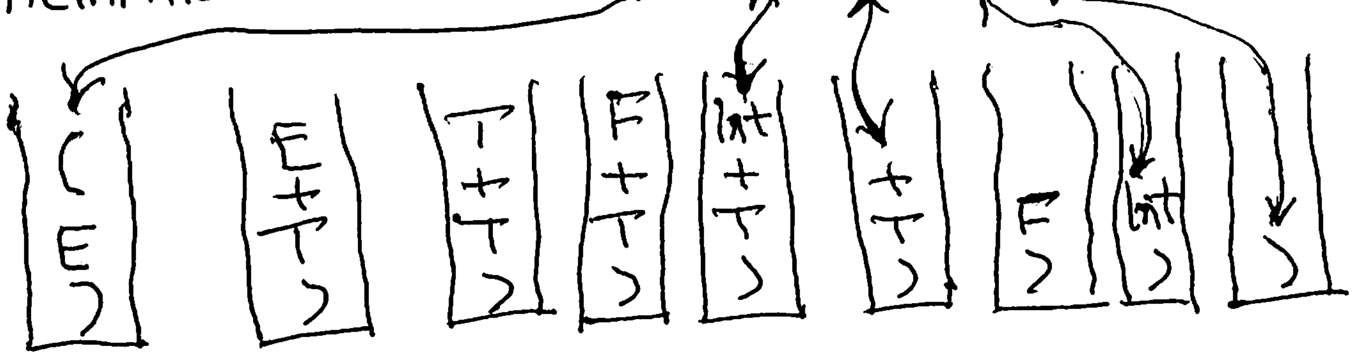
$T \rightarrow T * F \mid F$

$F \rightarrow (E) \mid Int$

INPUT: $7 * (3 + 21) \Rightarrow Int * (Int + Int)$



REMAINING INPUT: $(Int + Int)$



ACCEPTS BY EMPTY STACK

10/13/2016

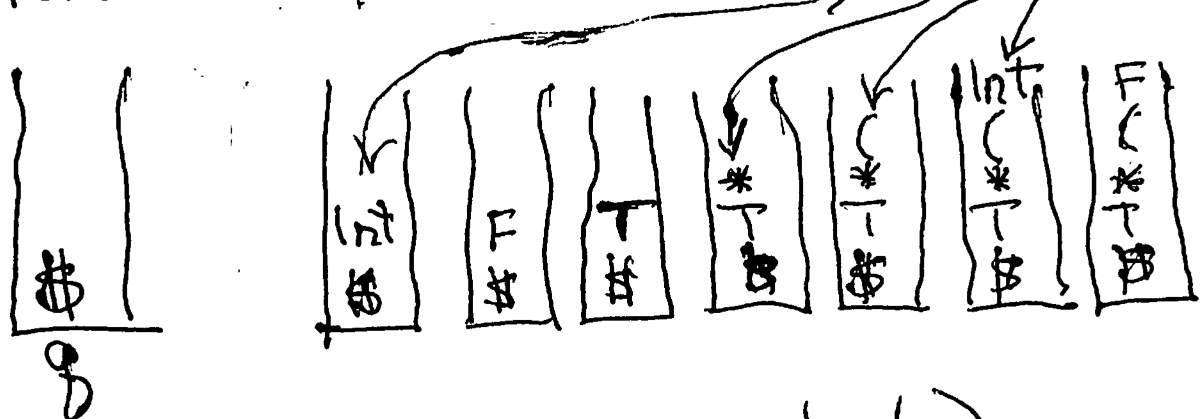
BOTTOM-UP PARSER

$E \to E + T \mid T$

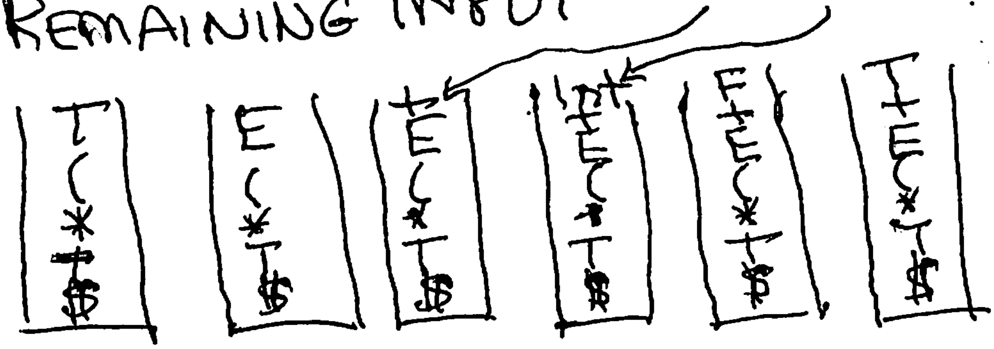
$T \to T * F \mid F$

$F \to (E) \mid Int$

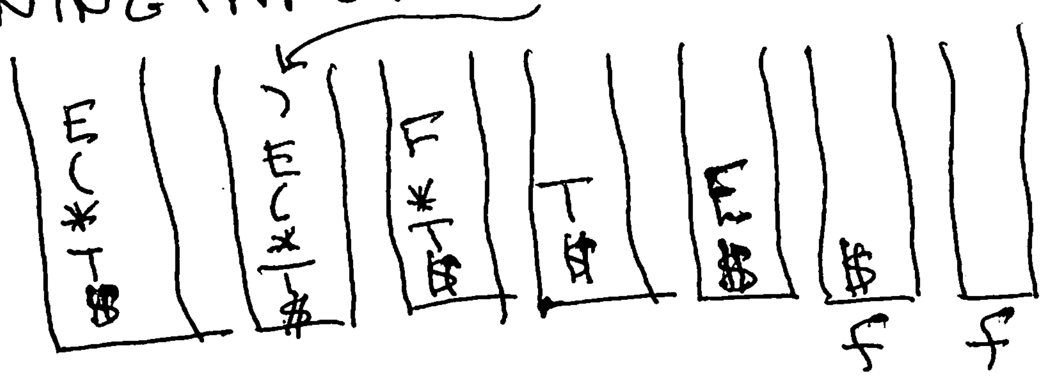
INPUT: 7*(3+21) ⇒ Int*(Int+Int)



REMAINING INPUT +Int)



REMAINING INPUT)



ACCEPTS BY FINAL STATE AND EMPTY STATE

OR JUST EMPTY STACK (CAN ALTER FOR FINAL)