10/4/2014

ONE MORE PROOF WRT CFGs, $G=(\{P\},\{0,1\},R,P)$

R: $P \to \lambda \mid 0 \mid 1 \mid 0P0 \mid 1P1$

$\mathcal{L}(G) = \{w \mid w$ IS A BINARY PALINDRONE$\}$

$= \{w \mid w \in \{0,1\}^* \;\&\; w = w^R\}$

PROOF: 1. ALL BINARY PALINDRONES ARE IN $\mathcal{L}(G)$

BASIS: $|w|=0$ OR $|w|=1$

$P \to \lambda$ SO $P \overset{*}{\Rightarrow} \lambda \Rightarrow \lambda \in \mathcal{L}(G)$

$P \to 0$ SO $P \overset{*}{\Rightarrow} 0 \Rightarrow 0 \in \mathcal{L}(G)$

$P \to 1$ SO $P \overset{*}{\Rightarrow} 1 \Rightarrow 1 \in \mathcal{L}(G)$

AND SO BASIS IS SHOWN

IH: ASSUME FOR ALL w, $|w| \le k$, $k \ge 1$,
$P \overset{*}{\Rightarrow} w$ WHENEVER $w = w^R$

IS: LET $|w| = k+1$. SINCE $k \ge 1$, $|w| \ge 2$ & SO
$w = 0x0$ OR $w = 1x1$ for $|x| = k-1$ AND $x = x^R$

BUT THEN
$P \Rightarrow 0P0 \overset{*}{\Rightarrow} 0x0$
OR $P \Rightarrow 1P1 \overset{*}{\Rightarrow} 1x1$

& SO $P \overset{*}{\Rightarrow} w$

10/4/2016

PROOF: 2 SHOW ~~ONLY~~ $\mathcal{L}(G)$ CONTAINS ONLY PALIDDRONES

DO INDUCTION OF # OF STEPS IN DERIVATION

BASIS: $k=1$

EITHER $P \Rightarrow \lambda$ OR $P \Rightarrow 0$ OR $P \Rightarrow 1$ AND

ALL ARE PALINDRONES

IH: ASSUME FOR ALL $i \le k$, $k \ge 1$,

IF $P \overset{i}{\Rightarrow} X$ ~~THEN~~ AND $X \in \{0,1\}^*$ THEN

X IS AN PALINDRONE

$$IS:\ \left.\begin{aligned} &P \Rightarrow 0P0 \overset{k}{\Rightarrow} 0X0 \\ &\text{OR } P \Rightarrow 1P1 \overset{k}{\Rightarrow} 1X0 \end{aligned}\right\} X \in \{0,1\}^*$$

ARE ONLY $k+1$ LENGTH DERIVATIONS

OF TERMINAL STRINGS.

SINCE $P \overset{k}{\Rightarrow} X$ IS A PALINDRONE

THEN SO ARE 0X0 AND 1X1

QED