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$L = \{w \mid w \in \{a,b\}^* \text{ \&}$
$w_a = w_b$

WHERE w_a IS # OF a's IN W; w_b IS # OF b's

$G = (\{S\}, \{a,b\}, S, P)$

$P: S \to aSbS \mid bSaS \mid \lambda$

PROVE $\mathcal{L}(G) = L$

FIRST, $\mathcal{L}(G) \subseteq L$

AS ALL RHS OF S HAVE SAME # OF a's AS b's

THUS, $w \in \mathcal{L}(G) \Rightarrow w \in L$

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$L \subseteq \mathcal{L}(G)$

INDUCTIVELY PROVE ON $|w|$
$w \in L \Rightarrow w \in \mathcal{L}(G)$

BASIS: $|w| = 0$
$S \to \lambda$ MEANS $S \Rightarrow \lambda$, SO
$\lambda \in L$ AND THAT IS ONLY
STRING OF LENGTH 0

IH: FOR ALL $i \leq k$, $k \geq 0$
IF $w \in L$, $|w| = 2i$ THEN $w \in \mathcal{L}(G)$

IS: SHOW FOR $|w| = 2k+2$
ASSUME w STARTS W AN a
THEN $w = a\alpha b\beta$
WHERE $\alpha \in L$, $\beta \in L$ & α IS
SHORTEST STRING LEADING
TO MATCH OF b FOR FIRST a
AS $|\alpha| \leq 2k$, $|\beta| \leq 2k$
$S \overset{*}{\Rightarrow} \alpha$, $S \overset{*}{\Rightarrow} \beta$ BY IH

SINCE $S \to aSbS \in P$

THEN

$S \Rightarrow aSbS \overset{*}{\Rightarrow} a\alpha bS$

$\overset{*}{\Rightarrow} a\alpha b\beta$

THEN $S \overset{*}{\Rightarrow} w$ + SO

$w \in \mathcal{L}(G)$

CAN REDO FOR STARTING WITH b, AND SO

$w \in L \Rightarrow w \in \mathcal{L}(G)$

COMBINING

$w \in L \Leftrightarrow w \in \mathcal{L}(G)$

AND SO

$L = \mathcal{L}(G)$

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④

$\{xy \mid |x|=|y|, x \neq y, x,y \in \{a,b\}^*\}$
$\cup \{w \mid |w| \text{ is ODD}, w \in \{a,b\}^*\}$

$= \overline{\{ww \mid w \in \{a,b\}^*\}}$

Odd LENGH IS EASY

$S_1 \rightarrow aT \mid bT$
$T \rightarrow aS_1 \mid bS_1 \mid \lambda$

OTHER PART IS CHALLENGE

IDEAS?

REMEMBER ONE TRANSCRIPTION ERROR DOES IT

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FORM IS

$x a y x' b y'$

OR

$x b y x' a y'$

$|x| = |x'|, |y| = |y'|$

BUT THERE IS ANOTHER WAY TO VIEW THIS

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HOW ABOUT JUST LOOK AT LENGTHS OF x, x', y, y' AND KEEP THOSE CONSTANT WHILE CHANGING POSITIONS BUT NOT LENGTHS OF PARTS?

$x a x' y b y'$

OR $x b x' y a y'$

$\underbrace{x' y}$ SAME LENGTH AS $y x'$

NOW CAN WRITE GRAMMAR FOR TRANSCRIPTION ERROR PART

$S_2 \to AB \mid BA$

$A \to XAX \mid a$

$B \to XBX \mid b$

$X \to a \mid b$

FINAL GRAMMAR START

$S \to S_1 \mid S_2$

Odd

MISMATCHED EVEN

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AMBIGUITY

$L = \{a^i b^j c^k \mid k > i \text{ or } k > j\}$

IS INHERENTLY AMBIGUOS

THINK ABOUT IT!

BUT ARITHMETIC EXPR.

ARE NOT

$E \to E + T \mid E - T \mid T$

$T \to T * F \mid T / F \mid F$

$F \to (E) \mid id \mid number$

DFA CAN RECOGNIZE id,
number, operators

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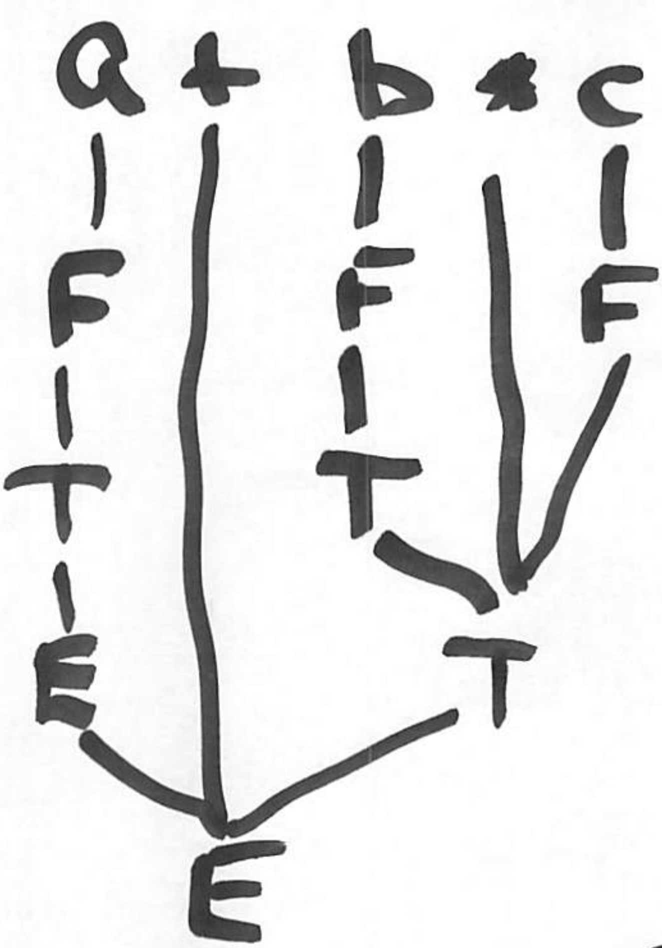

IS ONLY POSSIBLE PARSE
USING GRAMMAR ON 8

LEFTMOST

$E \Rightarrow E+T \Rightarrow T+T \Rightarrow F+T$
$\Rightarrow a+T \Rightarrow a+T*F \Rightarrow a+F*F$
$\Rightarrow a+b*F \Rightarrow a+b*c$

RIGTMOST

$E \Rightarrow E+T \Rightarrow E+T*F \Rightarrow E+T*c$
$\Rightarrow E+F*c \Rightarrow E+b*c$
$\Rightarrow T+b*c \Rightarrow F+b*c \Rightarrow a+b*c$

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A GRAMMAR IS AMBIGUOUS IFF THERE IS SOME STRING w IN LANGUAGE SUCH THAT

a. w IS THE YIELD OF TWO OR MORE DISTINCT PARSE TREES
b. w IS THE YIELD OF TWO OR MORE DISTINCT LM DERIV.
c. w IS THE YIELD OF TWO OR MORE DISTINCT RM DERIV.

LM = LEFTMOST
RM = RIGHTMOST

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PRACTICAL CONSIDERATIONS

TOP DOWN PARSER

ASSOC. WITH LM DERIV.

DOES NOT LIKE LEFT REC.

BOTTOM UP PARSER

ASSOC. WITH REVERSING
RM DERIV.

DOES NOT LIKE RIGHT REC.

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PARSERS AND GRAMMAR CLASSES (CFG SUBCLASSES)

LL(k)

INPUT L TO R

DERIV. L TO R

LR(k)

INPUT L TO R

DERIV. R TO L

k TOKEN LOOKAHEAD SUFFICIENT

LANGUAGES ASSOC. WITH LL(k) AND LR(k)

LR(1) LANG = DCFL

LR(1) $\subsetneq$ DCFG (UNAMB.)

LR(k+1) LANG = LR(k) LANG, $k \geq 1$

LR(k+1) GRAM $\supsetneq$ LR(k) GRAM, $k \geq 0$

LL(k+1) LANG/GRAM $\supsetneq$ LL(k) LANG/GRAM, $k \geq 0$

$\lim_{k \to \infty}$ LL(k) LANG = DCFL

HOWEVER, LL(1) GRAMMARS ARE USUALLY SUFFICIENT FOR SYNTAX OF PGM LANGUAGES EXCEPT FOR CONTEXT SENSITIVE PARTS, WHICH ARE HANDLED BY SYMBOL TABLES