9/15/16 REWRITING SYSTEMS ①

POST & THUE

THUE: ALPHABET Σ
FINITE BIDIRECTIONAL SET OF RULES
$\alpha_1 \Leftrightarrow \beta_1$
$\vdots$
$\alpha_k \Leftrightarrow \beta_k$

ANY STRING $\gamma\alpha_i\delta$ CAN BE CHANGED TO
$\gamma\alpha_i\delta \Leftrightarrow \gamma\beta_i\delta$
AND VICE VERSA

WE HAVE SYMMETRY
CAN DEFINE $\overset{*}{\Leftrightarrow}$
AS REFLEXIVE, SYM, TRANS. CLOSURE AND GET EQUIVALENCE CLASSES

9/15/16

POST:

THINK OF ONE-WAY REWRITING AS COMPUTATION STEPS, SO RULES ARE

$$\alpha_1 \to \beta_1$$
$$\vdots$$
$$\alpha_k \to \beta_k$$

$\xRightarrow{*}$ IS REFLEXIVE, TRANSITIVE CLOSURE OF $\gamma\alpha_i\delta \Rightarrow \gamma\beta_i\delta$ WHEN $\alpha_i \to \beta_i$

THIS IS DERIVATION NOT EQUIVALENCE

9/15/16

ENTER NOAM CHOMSKY

GRAMMITICAL ANALYSIS INVOLVES SYNTACTIC AND LEXICAL ITEMS.

SYNTAX IS A FORM OF CLASSIFICATION OR ABSTRACTION

E.G., <DECLARATIVE SENTENCE>
<NOUN PHRASE>
<VERB>

LEXICAL ITEMS ARE THE WORDS AND PUNCTUATION

SYNTACTIC ITEMS = VARIABLES
LEXICAL ITEMS = TERMINALS

9/15/16

FORMAL GRAMMAR

$G = (V, \Sigma, S, P)$

V: FINITE SET OF VARIABLES OR NON-TERMINALS

Σ: FINITE SET OF TERMINALS

S: $S \in V$ IS START SYMBOL

P: FINITE SET OF PRODUCTIONS AKA, RULES, EACH OF FORM

$\alpha \to \beta$, $\alpha \in (V \cup T)^* V (V \cup T)^*$

$\beta \in (V \cup T)^*$

MUST HAVE VARIABLE OF LHS

SINGLE STEP

$\gamma \alpha \delta \Rightarrow \gamma \beta \delta$ IF $\alpha \to \beta \in P$

DERIVATION IS $\overset{*}{\Rightarrow}$

$\mathcal{L}(G) = \{ w \mid w \in \Sigma^*, S \overset{*}{\Rightarrow} w \}$

9/15/16

HIERARCHY OF GRAMMARS

TYPE 0: PHRASE STRUCTURED (PSG)

NO CONSTRAINTS ON RULES

TYPE 1: CONTEXT SENSITIVE (CSG)

IF $\alpha \to \beta \in P$ THEN $|\alpha| \leq |\beta|$

TYPE 2: CONTEXT FREE (CFG)

IF $\alpha \to \beta \in P$ THEN $\alpha \in V$

i.e., $|\alpha| = 1$

TYPE 3: RIGHT LINEAR (REGULAR)

ALL RULES OF FORM

$A \to a \quad a \in \Sigma$

$A \to aB \quad a \in \Sigma, B \in V$

? WHAT ABOUT λ ?

WHAT IF $\lambda \in L$?

WE CAN ALLOW λ RULES IN REGULAR & CFG W/O CHANGING ACCEPTED LANGUAGES BEYOND ADDING λ TO L.

HOWEVER, FOR CSG, WE LIMIT λ TO RHS OF AN S-RULE AND THEN DO NOT ALLOW S ON ANY RHS

SOME TERMINOLOGY

IF $W \in (V \cup \Sigma)^*$, W IS A <u>SYNTACTIC FORM</u>

IF $W \in \Sigma^*$, W IS A <u>TERMINAL STRING</u>

TYPICALLY WE CALL THINGS SYNTACTIC FORMS WHEN THEY CAN BE ALTERED (CONTAIN A CHARACTER IN V)

EXTENDED FORM OF REGULAR

$A \rightarrow \alpha \quad , \alpha \in \Sigma^*$

$A \rightarrow \alpha B, \alpha \in \Sigma^*, B \in V$

HOWEVER, WE AVOID THE CASE OF

$A \rightarrow B \quad \alpha = \lambda$

FOR NOW, SO

$A \rightarrow \alpha B, \alpha \in \Sigma^+, B \in V$

EXTENDED NO STRONGER

CHANGE

$A \to a_1 a_2 \cdots a_k$, $k > 1$

TO $A \to a_1 \langle a_2 \cdots a_k \rangle$

WHERE $\langle a_2 \cdots a_k \rangle$ IS NEW

AND

$\langle a_2 \ldots a_k \rangle \to a_2 \ldots a_k$

APPLY ITERATIVELY UNTIL DOWN TO ONE TERMINAL

ALSO, CHANGE

$A \to a_1 a_2 \cdots a_k B$, $k > 1$

TO $A \to a_1 \langle a_2 \cdots a_k B \rangle$

& $\langle a_2 \cdots a_k B \rangle \to a_2 \ldots a_k B$

AND ITERATE

9/15/16

EXAMPLE REG. GRAMMAR

$(0+1)^* 11 (0+1)^*$

$G = (\{S, T, U\}, \{0, 1\}, S, P)$

$P: S \to 0S \mid 1S \mid 1T$

$T \to 1U$

$U \to 0U \mid 1U \mid \lambda$

- - - - - - - - - - - - - -

REMOVE λ-RULE

$P': S \to 0S \mid 1S \mid 1T$

$T \to 1U \mid 1$

$U \to \underline{0U} \mid \underline{1U} \mid \underline{0} \mid \underline{1}$

- - - - - - - - - - - - - -

CHALLENGE

$L = \{ wxw^R \mid w \in \{a,b\}^+, x \in \{a,b\}^* \}$

IS REGULAR!!!

9/15/16

CONVERT GRAMMAR TO NFA

$G = (V, \Sigma, S, P)$

$Q_G = (V', \Sigma, \delta_G, \{f\})$

$V' = V \cup \{f\}$

WHENEVER $A \to a$ $\quad a \in \Sigma \cup \{\lambda\}$

$\delta_G(A, a) \supseteq \{f\}$

WHENEVER $A \to aB$ $\quad B \in V$, $a \in \Sigma$

$\delta_G(A, a) \supseteq \{B\}$

δ_G CONTAINS NO OTHER TRANSITIONS

CLAIM:

$S \overset{*}{\Rightarrow} wX$, $w \in \Sigma^*$, $X \in V$

IFF

$\delta_G^*(S, w) \supseteq \{X\}$

&

$S \overset{*}{\Rightarrow} w$, $w \in \Sigma^*$

IFF

$\delta_G^*(S, w) \supseteq \{f\}$

9/15/16

CONVERT DFA
TO REG. GRAMMAR

$Q = (Q, \Sigma, \delta, q_0, F)$

$G_Q = (Q, \Sigma, q_0, P_Q)$

WHENEVER $\delta(p, a) = q$

$p \to aq \in P_Q$

WHENEVER $f \in F$

$f \to \lambda \in P_Q$

P_Q CONTAINS NO OTHER RULES

Claim:

$$\delta^*(q_0, w) = t, \; w \in \Sigma^*, \; t \in Q$$

IFF

$$q_0 \overset{*}{\underset{G_Q}{\Rightarrow}} wt$$

&

$$\cancel{q_0 \underset{G_Q}{\Rightarrow}} \; \delta^*(q_0, w) \in F$$

IFF

$$q_0 \overset{*}{\underset{G_Q}{\Rightarrow}} w$$