1/6/16

Q, R, P SETS OF STRINGS
REALLY THEY ARE REG. EXPR. FOR REG. SETS

$$R = Q + RP$$

WE ASSUME $\lambda \notin P$

WHY?

LET'S TRY $P = \{\lambda\}$ OR $P = \lambda$

GET

$$R = Q + R$$

IF Q IS COUNTABLY INFINITE,

HOW MANY SOLUTIONS ARE THERE TO ABOVE?

Q IS <u>A</u> SOLUTION, BUT SO IS $Q + X$, AND THIS IS DISTINCT IF $X \cap Q = \emptyset$.

$R = Q + X$ ✓ $R = Q + (Q + X) = Q + X$ ✓

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Q, R, P SETS OF STRINGS

$R = Q + RP$

WE ASSUME $\lambda \notin P$

WHY ?

LET'S TRY $P = \{\lambda\}$ OR $P = \lambda$

GET

$R = Q + R$

ASSUME $Q \neq \Sigma^*$ ($R = \Sigma^* + R$ IS $R = \Sigma^*$)

ASSUME $Q \neq \emptyset$ ($R = \emptyset + R$ IS $R = R$!!)

TO BE INTERESTING, ASSUME Q IS CO-INFINITE ($\Sigma^* - Q$ IS INFINITE)

CLEARLY Q IS A SOLUTION

$Q = R = Q + Q = Q$ ✓

BUT, SO IS $Q + X$, $X \cap Q = \emptyset$

$Q + X = R = Q + Q + X = Q + X$ ✓

THERE ARE AN ~~[illegible]~~ UNCOUNTABLY INFINITE NUMBER OF SOLUTIONS

9/16/14 SUBSTITUTION USING REG. EXPR. 3

$f(a) = L_a \quad a \in \Sigma$

WHERE L_a IS REGULAR SET ASSOCIATED WITH REGULAR EXPRESSION r_a

$f(\lambda) = \lambda \quad f(\phi) = \phi$

$$\left.\begin{aligned} f(xa) &= f(x) \cdot L_a \\ &\text{OR} \\ f(ax) &= L_a \cdot f(x) \end{aligned}\right\} a \in \Sigma$$

$f(L) = \{ f(x) \mid x \in L \}$

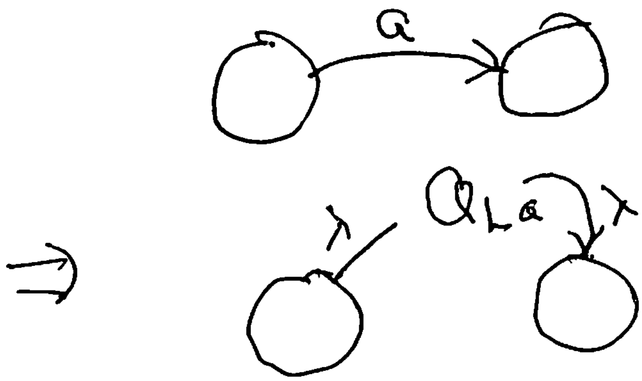
IF L IS REGULAR, $\exists$ A REG. EXPRESSION $r_L \ni L$ IS THE REGULAR SET ASSOC. WITH L

CAN GET REG. EXPR. FOR $f(L)$ BY CHANGING EACH ATOM IN r_L FROM

$f(\lambda) = \lambda$; $f(\phi) = \phi$

$f(a) = r_a$

9/6/16 SUBSTITUTION USING NFAs



9/[redacted]/15 [redacted]

$f(a) = \{a, a'\}$

$g(a) = a'$

$h(a) = a$

$h(a') = \lambda$

SUBSTRING$(L) =$

$$h(f(L) \cap g(\Sigma^*)\Sigma^* g(\Sigma^*))$$

PREFIX$(L) =$

$$h(f(L) \cap \Sigma^* g(\Sigma^*))$$

SUFFIX$(L) =$

$$h(f(L) \cap g(\Sigma^*)\Sigma^*)$$

$$L/R = h(f(L) \cap \Sigma^* g(R))$$

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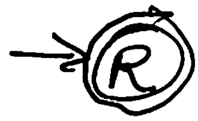
1. CONSIDER NFA  OVER $\Sigma = \{a\}$

THIS ACCEPTS $\{\lambda\}$

USING REG. EQS., $R = \lambda$ ✓

WHAT IF? 

USING REG. EQS. $R = \lambda + R \cdot \lambda$

CLEARLY $R = \lambda \cdot \lambda^* = \lambda$ IS A SOLUTION

BUT, SO IS $R = \lambda + a$ AS

$$\lambda + a = \lambda + (\lambda + a) \cdot \lambda = \lambda + \lambda^2 + a = \lambda + a$$

IN FACT, IF $R \subseteq a^*$ THEN R IS A SOLUTION,

SO THIS HAS AN UNCOUNTABLE NUMBER OF SOLUTIONS

2. MINIMIZATION

FIRST, CONSIDER ONLY DFAs WHERE ALL STATES ARE REACHABLE FROM (q_0). CALL DFA $Q = (Q, \ldots)$

CLEARLY, IF $\delta^*(q_0, x) = \delta^*(q_0, y)$ THEN

$$\forall z \; \delta^*(q_0, xz) = \delta^*(q_0, yz)$$

IF WE DEFINE R_Q BY

$$x R_Q y \text{ IFF } \delta^*(q_0, x) = \delta^*(q_0, y)$$

THEN R_Q IS AN EQUIV. RELATION THAT PARTITIONS Σ^* INTO $|Q|$ EQUIV. CLASSES AND THE UNION OF SOME OF THESE CLASSES DEFINES $\mathcal{L}(Q)$ -- CLASSES ASSOC. WITH F

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3. WE CAN DEFINE STATES OF $\mathcal{Q}$, P, Q AS INDISTINGUISHLE IF ~~[illegible]~~ AND $\forall z\,[\delta^*(p,z) \in F \Leftrightarrow \delta^*(q,z) \in F]$ 2

IF TWO STATES ARE INDISTINGUISHABLE THEY HAVE THE SAME FUTERES AND SO CAN BE MERGED.

INDISTINUISHABLE STATE MERGING CREATES A NEW, LESS REFINED, EQUIVALENCE RELATION OVER Σ^* THAT STILL HAS THE PROPERTY THAT $\mathcal{L}(\mathcal{Q})$ IS THE UNION OF SOME CLASSES, BUT THE NUMBER OF CLASSES IS SMALLER (NO LARGER) THAN $|F|$

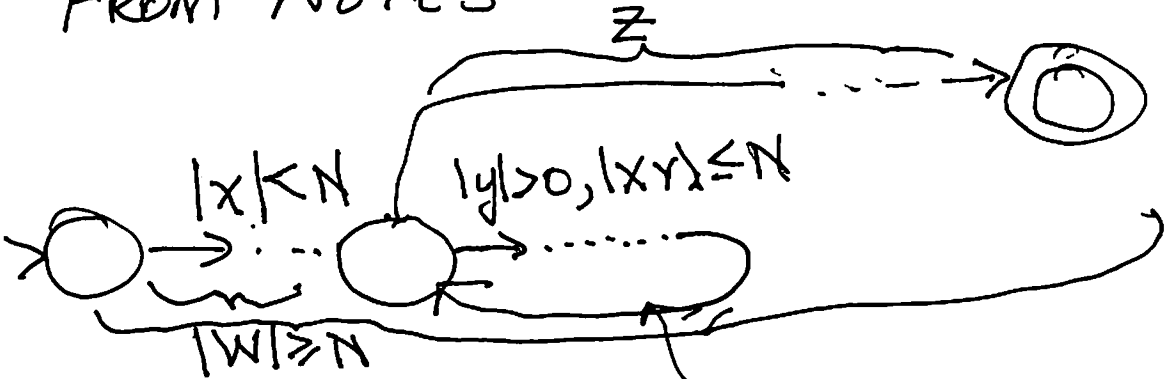
NEW FINAL STATES ARE ASSOCIATED WITH EQUIV. CLASSES CONTAINING STRINGS THAT LED TO FINAL STATES IN $\mathcal{Q}$

4. WE ACTUALLY ATTACK MINIMIZATION BY DISCOVERING DISTINGUISHABLE STATES
THE BEST APPROACH IS VIA A LOWER TRIANGULAR MATRIX

BACK TO POWER POINT

5. PUMPING LEMMA

FROM NOTES



$|w| \geq N$
$|Q| = N$

$w = xyz \in L$

CAN REPEAT THIS PART 0 OR MORE TIMES, SO

$xy^i z \in L, \forall i \geq 0$

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$L = \{ww^R \mid w \in \{a,b\}^*\}$

P.L. GIVE ME N

ME: $a^N bba^N$

P.L. $a^N bba^N = xyz$

$|xy| \leq N \quad |y| > 0$

ME: $i = 0$

$a^{N-|y|} bba^N \notin L$

P.L. $\forall i \geq 0 \quad xy^iz \in L$

X

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$L = \{ww^R \mid w \in \{a,b\}^*\}$

ASSUME REGULAR

$[a^i]$ $i \geq 0$

$a^i bba^i \in L$

~~$a^i bb$~~

$a^j bba^i \notin L$ $i \neq j$

$[a^i] \neq [a^j]$ $i \neq j$