8/31/16 COMPLEMENT OF REGULAR LANGUAGE

1. REALLY BEGS FOR DFA VS NFA

$\mathcal{Q} = (Q, \Sigma, \delta, q_0, F)$ A DFA

$\mathcal{L}(\mathcal{Q}) = \{w \mid \delta^*(q_0, w) \in F\}$

$\overline{\mathcal{L}(\mathcal{Q})} = \{w \mid \delta^*(q_0, w) \notin F\}$

$= \{w \mid \delta^*(q_0, w) \in Q - F\}$

Thus

$\overline{\mathcal{L}(\mathcal{Q})} = \mathcal{L}(\mathcal{Q}^c)$ WHERE

$\mathcal{Q}^c = (Q, \Sigma, \delta, q_0, Q - F)$

WHICH IS A DFA

2. $\mathcal{Q} = (Q, \Sigma, \delta, q_0, F)$ AN NFA

$\mathcal{L}(\mathcal{Q}) = \{w \mid \delta^*(q_0, w) \cap F \neq \emptyset\}$

BUT $\mathcal{L}(\mathcal{Q}') = \{w \mid \delta^*(q_0, w) \cap (Q - F) \neq \emptyset\}$

IS NOT $\overline{\mathcal{L}(\mathcal{Q})}$

8/31/16 WHEN SHOWING CLOSURE, WANT RIGHT MODEL

3. UNION

FOR NFA'S

$\mathcal{Q}_1 = (Q_1, \Sigma, \delta_1, q_{10}, F_1)$

$\mathcal{Q}_2 = (Q_2, \Sigma, \delta_2, q_{20}, F_2)$

CAN DO UNION OF $L(\mathcal{Q}_1)$ AND $L(\mathcal{Q}_2)$

BY

$\mathcal{Q}_3 = (\{q_0\} \cup Q_1 \cup Q_2, \delta_3, q_0, F_1 \cup F_2)$

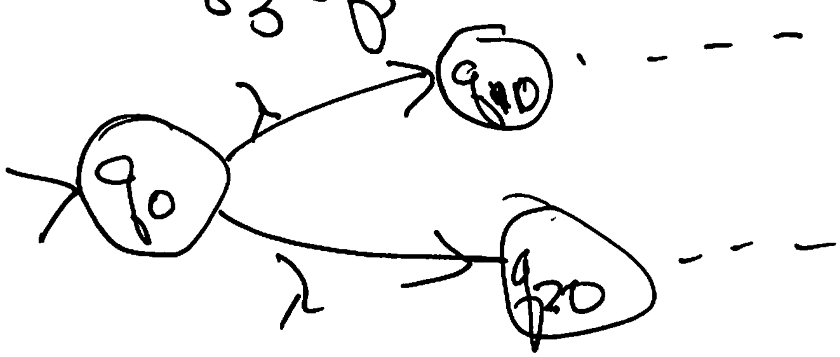
WHERE

$\delta_3(q_0, \lambda) = \{q_{10}, q_{20}\}$

$\delta_3(q, a) = \delta_1(q, a)$ WHEN $q \in Q_1$

$\delta_3(q, a) = \delta_2(q, a)$ WHEN $q \in Q_2$

$\delta_3(q, a)$




// $\mathcal{Q}_1$

// $\mathcal{Q}_2$

4. UNION CAN BE DONE WITH DFA BY

$Q_1 = (Q_1, \Sigma, \delta_1, q_{10}, F_1)$ ; $Q_2 = (Q_2, \Sigma, \delta_2, q_{20}, F_2)$

$Q_3 = (Q_1 \times Q_2, \Sigma, \delta_3, \langle q_{10}, q_{20} \rangle, F_3)$

WHERE

$Q_1 \times Q_2 = \{ \langle p, q \rangle \mid p \in Q_1, q \in Q_2 \}$

$\delta_3(\langle p, q \rangle, a) = \langle \delta_1(p, a), \delta_2(q, a) \rangle$

SYNCHRONIZED PARALLEL (MISD - MULTIPLE INSTRUCTION, SINGLE DATA)

$F_3 = F_1 \times Q_2 \cup Q_1 \times F_2$

ACCEPTS IF FIRST OR SECOND OR BOTH ACCEPT

5. CAN USE THIS CONSTRUCT FOR MANY SET OPERATIONS, SOME OF WHICH, LIKE COMPLEMENT, CANNOT BE DIRECTLY PROVEN WITH NFA MODEL

$\mathcal{L}(Q_1) \cap \mathcal{L}(Q_2)$ SET $F_3 = F_1 \times F_2$

$\mathcal{L}(Q_1) - \mathcal{L}(Q_2)$ SET $F_3 = F_1 \times (Q_2 - F_2)$

$\mathcal{L}(Q_1) \oplus \mathcal{L}(Q_2)$ SET $F_3 = F_1 \times (Q_2 - F_2) \cup (Q_1 - F_1) \times Q_2$

$= F_1 \times Q_1 \cup Q_1 \times F_2 - F_1 \times F_2$

6. OF COURSE, IF YOU HAVE UNION & COMPLEMENT, YOU CAN BUILD ALL OF ABOVE, BUT IT'S NOT AS INSTRUCTIVE OR INTUITIVE AS CONSTRCTION

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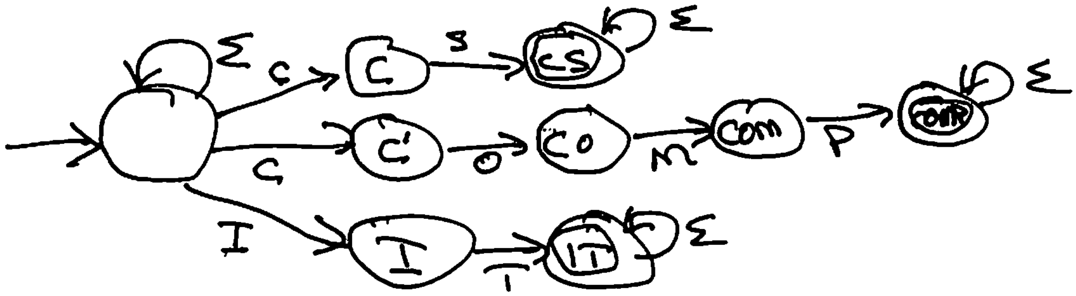
7. NFA'S

IF WANT ANY STRING CONTAINING "CS" OR "COMP" OR "IT"




GENEALLY EASIER TO WRITE NFA WHEN MULTIPLE PATHWAYS TO SUCCESS.

8. CONVERSION TO DFA

FIRST WANT TO ACCOMMODATE $\lambda$-TRANSITIONS

FOR $q \in Q$,

$$\lambda\text{-CLOSURE}(q) = \{t \mid \delta^*(q,\lambda) \supseteq \{t\}\}$$
$$= \{t \mid t \in \delta^*(q,\lambda)\}$$

FOR $S \subseteq Q$

$$\lambda\text{-CLOSURE}(S) = \{t \mid t \in \bigcup_{q \in S} \lambda\text{-CLOSURE}(q)\}$$

LET $A = (Q, \Sigma, \delta, q_0, F)$ BE AN NFA

DEFINE

$$A' = (\langle P(Q)\rangle, \Sigma, \delta', \langle\lambda\text{-CLOSURE}(q_0)\rangle, F')$$

WHERE

$$\delta'(\langle S\rangle, a) = \langle \lambda\text{-CLOSURE}(\delta(S,a))\rangle$$
$$= \langle \bigcup_{q \in S} \lambda\text{-CLOSURE}(\delta(q,a))\rangle, a \in \Sigma, S \in P(Q)$$

$$F' = \{\langle S\rangle \in \langle P(Q)\rangle \mid (S \cap F) \neq \emptyset\}$$

THIS LEADS TO $2^{|Q|}$ STATES, MOST OF WHICH ARE UNREACHABLE FROM START.

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# 9. COMPACT DFA FROM NFA

BUILD NEEDED SUBSETS FROM STATES ACCESSIBLE FROM START

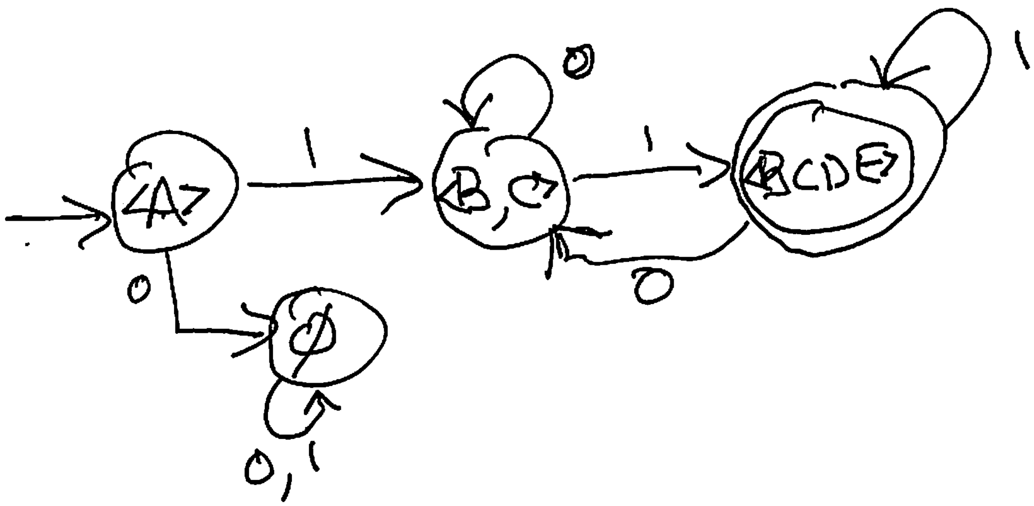
PROCESS
COMPUTE $\lambda$-CLOSURE OF EACH $q \in Q$

START WITH $\lambda$-CLOSURE OF $q_0$ AND COMPUTE ALL SUCCESSOR STATES OF ONE STEP, ALWAYS DOING $\lambda$-CLOSURE OF THESE STATE SETS.

NEVER ADD AN UNNEEDED STATE

NOTE: $\langle \phi \rangle$ IS COMMON AND IS "DEAD" STATE

EVEN THIS DOES NOT GET MINIMAL STATE, BUT IS GOOD START



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