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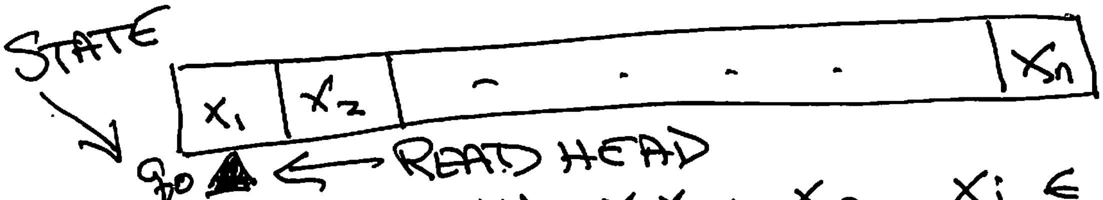
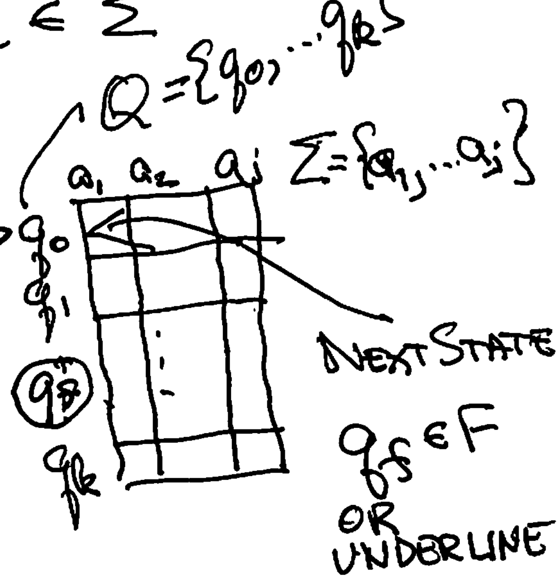
FINITE STATE AUTOMATON
DFA vs NFA



INPUT IS $w = x_1 x_2 \cdots x_n$, $x_i \in \Sigma$

$q_0 \in Q$ IS START STATE



ACTIONS ARE:

- LOOK UP IN STATE TABLE
  ENTITY IS AT ROW q_m COL a_i
  WHERE STATE IS q_m
  INPUT SYMBOL IS a_i
- CHANGE STATE TO ENTITY
  AT q_m, a_i
- MOVE READ HEAD ONE CELL ON TAPE
- IF FALL OFF END THEN DONE
  ACCEPT IF IN ACCEPTING STATE, $p \in F \subseteq Q$
  REJECT OTHERWISE
- IF NOT OFF END (MORE TO READ),
  ITERATE PROCESS

DFA HAS ONE STATE FOR EACH PAIR q_m, a_i

MAPPING IS $\delta : Q \times \Sigma \to Q$

PROCESS ALWAYS HALTS WITH ACCEPT OR REJECT

8/26/16 FORMAL MODEL OF DFA

$$A = (Q, \Sigma, \delta, q_0, F)$$

Q IS FINITE SET OF STATES

Σ IS FINITE ALPHABET FOR INPUT STRINGS

δ IS TRANSITION (NEXT STATE) FUNCTION

$\delta : Q \times \Sigma \to Q$

$q_0 \in Q$ IS ~~START~~ STATE

$F \subseteq Q$ IS SET OF FINAL (ACCEPTING) STATES

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TRANSITIONS AND LANGUAGE

$\delta^*: Q \times \Sigma^* \to Q$ IS REFLEXIVE, TRANSITIVE CLOSURE OF δ

$\delta^*(q, \lambda) = q$

$\delta^*(q, ax) = \delta^*(\delta(q, a), x) \quad a \in \Sigma, x \in \Sigma^*$

NOTE: $\delta^*(q, a) = \delta(q, a)$

$\delta^+(q, w) = \delta^*(q, w)$ WHEN $|w| > 0$

IF $A = (Q, \Sigma, \delta, q_0, F)$ THEN

$\mathcal{L}(A) = \{ w \mid \delta^*(q_0, w) \in F \}$

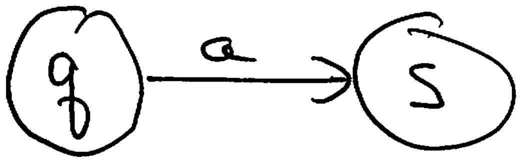
$\mathcal{L}(A)$ IS CALLED ~~AN~~ FINITE STATE OR A REGULAR LANGUAGE

THE CLASS OF REGULAR LANGUAGES IS DEFINED AS THE SET OF ALL LANGUAGES ACCEPTED BY DFAs.

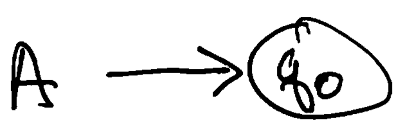
TRANSTION TABLES VS STATE DIAGRAMS

TRANSITION TABLES ARE GREAT FOR COMPUTERS AND AWFUL FOR HUMANS

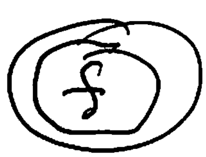
STATE DIAGRAMS ARE ROOTED GRAPHS THAT HAVE DIRECTED ARCS FROM STATE q TO STATE s, LABELED a, WHENEVER $\delta(q, a) = s$



THE ROOT IS THE START STATE AND MUST HAVE AN ENTERING ARC WITH NO PRIOR NODE (CAN THINK OF ITS BEING LABELED WITH λ



ALL FINAL STATES HAVE A DOUBLE CIRCLE



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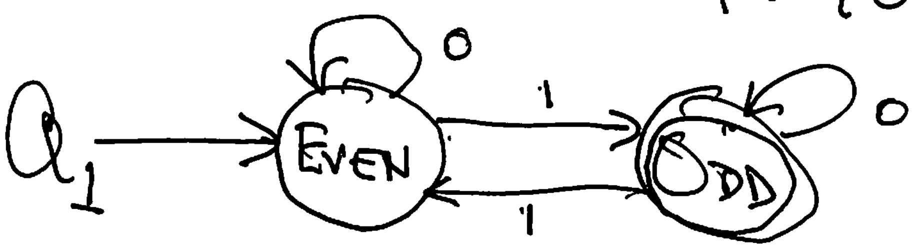
SAMPLES

ODD PARITY

$\Sigma = \{0,1\}$ $Q = \{\text{EVEN}, \text{ODD}\}$
$F = \{\text{ODD}\}$ $q_0 = \text{EVEN}$

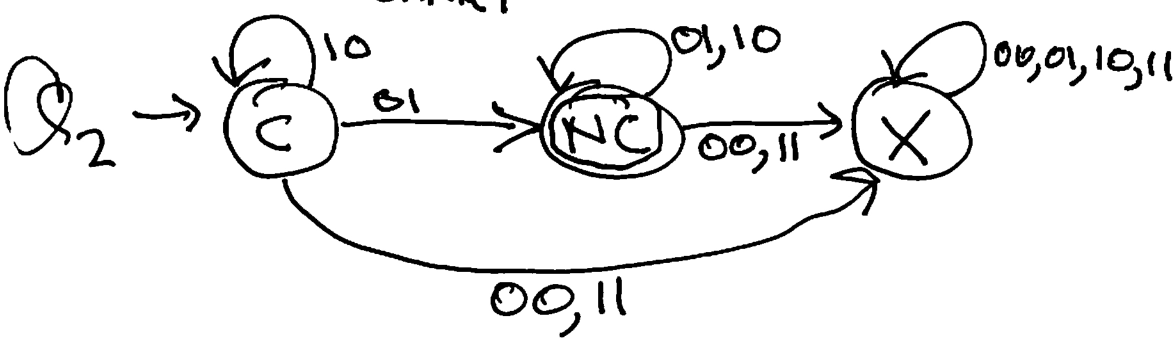


CHECK FOR 2'S COMPLEMENT

$\Sigma = \{0,1\} \times \{0,1\} = \{00, 01, 10, 11\}$

THINK OF THIS AS TWO SYNCHRONIZED CHANNELS OF INPUT (TOP IS #, BOTTOM IS 2'S COMPL.) ASSUME DATA COMES ~~[illegible]~~ MOST SIGNIFICANT TO LEAST SIGNIFICANT BIT (LEFT TO RIGHT) → THINK OF NEW BIT AS SHIFT LEFT THEN ADD, OR MULTIPLY BY 2 AND ADD

$Q = \{C, NC, X\}$ — NO CARRY (NC), REJECT (X), CARRY (C) $\quad q_0 = C$
$F = \{NC\}$



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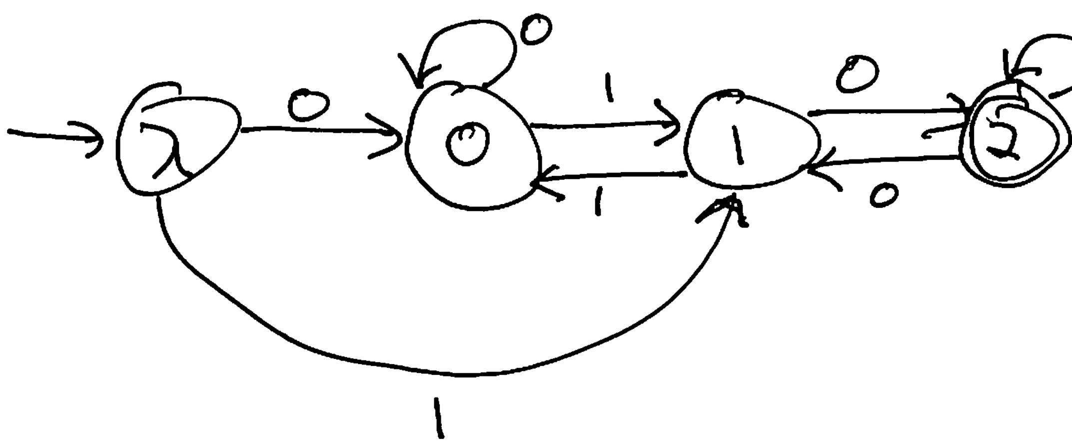
ANOTHER SAMPLE

ACCEPT BINARY STRING (MSB TO LSB) THAT, WHEN INTERPRETED AS DECIMAL NUMBERS, ARE 2 MOD 3

$Q = \{\lambda, 0, 1, 2\}$ $q_0 = \lambda$ $F = \{2\}$

$\Sigma = \{0, 1\}$

A_3



NOTE: $2*0+0=0$

$2*0+1=1$

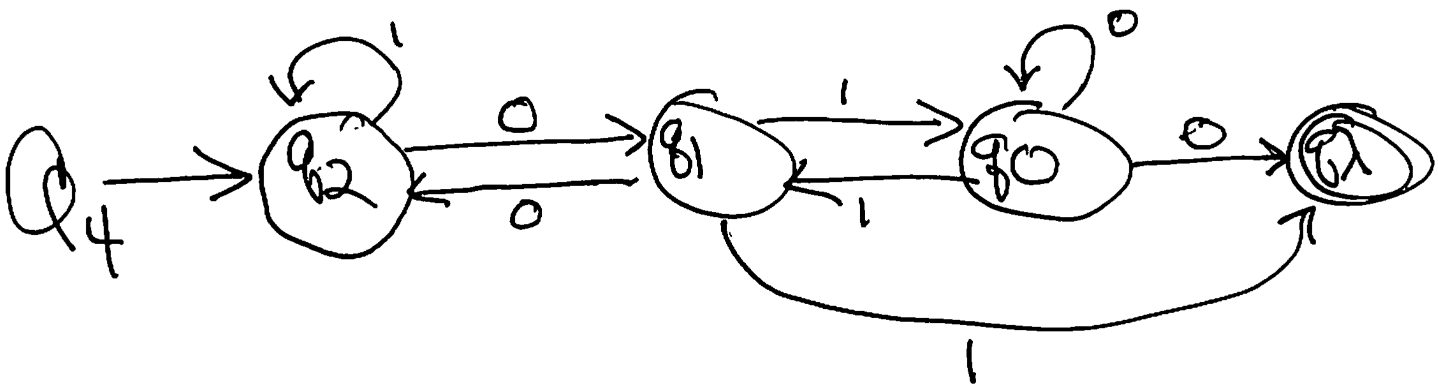
$2*1+0=2$

$2*1+1=3=0 \bmod 3$

$2*2+0=4=1 \bmod 3$

$2*2+1=5=2 \bmod 3$

8/26/11 SAMPLE Q_3 READ LSB TO MSB



THIS REVERSES EARLIER MACHINE

(WILL SHOW REVERSE OF REGULAR IS REGULAR IN A BIT)

BUT NOW WE ARE NON-DETERMINTIC

$$\delta(q_1, 1) = \{q_0, q_\lambda\}$$
$$\delta(q_\lambda, 0) = \delta(q_\lambda, 1) = \{\}$$
$$\delta(q_0, 0) = \{q_0, q_\lambda\}$$

8/26/16 NON-DETERMINISTIC AUTOMATON (NFA)

REALLY VIEW AS $\{q_0\}$

$$Q = (Q, \Sigma, \delta, q_0, F)$$

$$\delta : Q \times \Sigma_\epsilon \rightarrow 2^Q = P(Q)$$

$$\Sigma_\epsilon = \Sigma \cup \{\epsilon\} = \Sigma \cup \{\lambda\}$$

$$\lambda\text{-CLOSURE}(S) = \{t \mid t \in \delta^*(S, \lambda)\},\ S \in P(Q)$$

HERE, WE USED EXTENDED δ THAT OPERATES ON SETS $\delta(S,a) = \bigcup_{q \in S} \delta(q,a)$ WHERE $a \in \Sigma_\epsilon$

THIS ACCOMMADETES STATE CHANGES WITHOUT READING ANY INPUT.

$$\delta^*(S, \lambda) = \lambda\text{-CLOSURE}(S)$$

$$\delta^*(S, ax) = \bigcup_{q \in S} \bigcup_{p \in \text{CLOSURE}(\delta(q,a))} \delta^*(p, x),\quad a \in \Sigma,\ x \in \Sigma^*,\ S \in P(Q)$$

δ^+ AS BEFORE

$$L(Q) = \{w \mid \delta^*(\{q_0\}, w) \cap F \neq \emptyset\}$$

ACTUALLY, BETTER TO USE

$$L(Q) = \{w \mid \delta^*(\lambda\text{-CLOSURE}(\{q_0\}), w) \cap F \neq \emptyset\}$$

OR EVEN CHANGE START TO A SET $\lambda\text{-CLOSURE}(\{q_0\})$

8/26/6 CONSIDER LEXICAL ANALYSIS

ASSUME FIRST CHARACTER READ IN ADVANCE