COT 4210 Fall 2016 Sample Midterm 2 Name: ________________

1. Assuming you have computed the sets, $\mathbf{R^{k}_{i,j}}$, for each pair of states, $\mathbf{(q_i,q_j)}$, in some DFA. How is $\mathbf{R^{k+1}_{i,j}}$ calculated, where **k+1** is no greater than the number of states in the associated DFA?

2. Write a **Context Free Grammar** for the language
$\mathbf{L = \{ a^k\ b^m\ c^n \mid k = n + m,\ or\ m = k + n,\ or\ n = k + m,\ k>0,\ m>0,\ n>0 \}}$.

3. Use Myhill-Nerode to show that the language of Problem 2 is not Regular. That language is
$\mathbf{L = \{ a^k\ b^m\ c^n \mid k = n + m,\ or\ m = k + n,\ or\ n = k + m,\ k>0,\ m>0,\ n>0 \}}$.

4. Consider the language

L = { $a^n b^{n!}$ | n>0 }.

Use the Pumping Lemma for Context-Free Languages to show that **L** is not context-free.

5. Present the **CKY** recognition matrix for the string **bbabb** assuming the Chomsky Normal Form grammar, **G = ({S,A,B,C,D }, {a,b}, R, S)**, specified by the rules **R**:

S → AB | BA | BD
A → CS | CD | a
B → DS | b
C → a
D → b

	b	**b**	**a**	**b**	**b**
1					
2					
3					
4					
5					

6. Choosing from among **(Y) yes**, **(N) No**, categorize each of the following closure properties. No proofs are required.

Problem / Language Class (C)	**Regular**	**Context Free**
Closed under union with Context Free languages?		
Closed under quotient with languages of its own class (C), i.e., L1/L2		
Closed under difference with languages of its own class (C), i.e., (difference (L1, L2) = L1 – L2)?		
Closed under intersection with Context Free languages?		

7. Prove that any class of languages, ***C***, closed under union, concatenation, intersection with regular languages, homomorphism and substitution (e.g., the Context-Free Languages) is closed under **Erase Middle with Regular Sets (em)**, where **L** $\in$ ***C***, **R** is Regular, **L** and **R** are over the alphabet **Σ**, and **L em R = { xz | x,y** $\in$ **Σ**$^{+}$ and **∃y** $\in$ **R,** such that **xyz** $\in$ **L }.** You may assume substitution **f(a) = {a, a'},** and homomorphisms **g(a) = a'** and **h(a) = a, h(a') = λ**. Here **a∈Σ** and **a'** is a distinct new character associated with each **a∈Σ**.
You must be very explicit, describing what is produced by each transformation you apply.

8. Consider the CFG **G = ({ S, T }, { a, b }, R, S)** where **R** is:

S → a T T | T S | a

T → b S T | b

a.) Present a pushdown automaton that accepts the language generated by this grammar. You may (and are encouraged) to use a transition diagram where transitions have arcs with labels of form **a,** $\alpha \rightarrow \beta$ where **a** $\in \Sigma \cup \{\lambda\}$, $\alpha, \beta \in \Gamma^*$. Note: I am encouraging you to use extended stack operations.

What parsing technique are you using? (Circle one) **top-down** or **bottom-up**

How does your PDA accept? (Circle one) **final state** or **empty stack** or **final state and empty stack**

What is the **initial state**? __________________________

What is the **initial stack content**? __________________________

What are your **final states** (if any)? __________________________

b.) Now, using the notation of **ID**s (Instantaneous Descriptions, **[q, x, z]**), describe how your PDA accepts strings generated by **G**.

9. Consider the context-free grammar **G = ({ S, A, B }, { a,b }, R, S)**, where **R** is:

S → SAB | BA
A → AB | a
B → bS | b | λ

a.) Remove all **λ**-rules from **G**, creating an equivalent grammar **G**'. Show all rules.

b.) Remove all **unit** rules from **G'**, creating an equivalent grammar **G**''. Show all rules.

c.) Convert grammar **G''** to its **Chomsky Normal Form** equivalent, **G'''**. Show all rules.