Assignment # 7.1a Key

1. a.) Use the Pumping Lemma for CFLs to prove that the following is NOT a CFL
   L = { a^i b^j | j > 2*I }

   This language is a CFL, so you cannot show it to be a non-CFL. A grammar that works is

   S → aSbb | Sb | b

Assignment # 7.1b Key

1. b.) Use the Pumping Lemma for CFLs to prove that the following is NOT a CFL.

$L = \{ a^n b^{Fib(n)} \mid n>0 \}$, where **Fib(i)** is the **i^{th}** Fibonacci number

PL: Provides N>0

We: Choose $a^N b^{N!} \in L$

PL: Splits $a^N b^{N!}$ into uvwxy, $|vwx| \leq N$, $|vx| > 0$, such that $\forall i \geq 0\ uv^i wx^i y \in L$

We: Choose i=2

Case 1: vwx contains only b's, then we are increasing the number of b's while leaving the number of a's unchanged. In this case uv^2wx^2y is of form $a^N b^{N!+c}$, c>0 and this is not in L.

Case 2: vwx contains some a's and maybe some b's. Under this circumstances uv^2wx^2y has at least N+1 a's and at most N!+N-1 b's. But $(N+1)! = N!(N+1) = N!*N+N \geq N! + N > N!+N-1$ and so is not in L.

Cases 1 and 2 cover all possible situations, so L is not a CFL

Assignment # 7.2

2. Present the **CKY** recognition matrix for the string **babba** assuming the Chomsky Normal Form grammar,
   G = ({S,A,B,C,D }, {a,b}, R, S), specified by the rules **R**:

S → AB | BA | SC

A → CS | CD | a

B → DS | b

C → a

D → b

	b	a	b	b	a
1	B,D	A,C	B,D	B,D	A,C
2	S	S,A		S	
3	S,B	S	B		
4	B	S			
5	B,S				

Assignment # 7.3a

3. Consider the context-free grammar **G = ({ S } , { a , b } , S , P),** where **P** is:

   $\mathbf{S} \rightarrow \mathbf{S\,a\,S\,b\,S} \mid \mathbf{S\,b\,S\,a\,S} \mid \mathbf{S\,a\,S\,a\,S} \mid \mathbf{a} \mid \lambda$

   Provide the first part of the proof that

   L (G) = L = { w | w has at least as many **a's** as **b's }**

   That is, show that $\mathbf{L(G)} \subseteq \mathbf{L}$

   To attack this problem we can first introduce the notation that, for a syntactic form α, α_a = the number of **a's** in α, and α_b = the number of **b's** in α. Using this, we show that if $\mathbf{S} \Rightarrow^* \alpha$, then $\alpha_b \leq \alpha_a$ and hence that $\mathbf{L(G)} \subseteq \mathbf{L}$:

   A straightforward approach is to show, inductively on the number of steps, **i**, in a derivation, that, if $\mathbf{S} \Rightarrow^{i} \alpha$, then $\alpha_b \leq \alpha_a$.

Assignment # 7.3b

Basis (i=1): Since S $\Rightarrow \alpha$ iff S $\rightarrow$ α and all rhs of S have $\alpha_b \leq \alpha_a$ then the base case holds

IH: Assume if S $\Rightarrow_m \alpha$, then $\alpha_b \leq \alpha_a$, whenever $m \leq n$

IS: Show that if S $\Rightarrow_{n+1} \alpha$, then $\alpha_b \leq \alpha_a$

If S α then S $\Rightarrow_n$ β and $\beta \Rightarrow \alpha$

Since G has only one non-terminal S, the rewriting of β to α involves a single application of one of the S-rules. By the I.H., β has the property that $\beta_b \leq \beta_a$. Since a single application of an S rule either adds no b's or a's, one a, one a and one b, or two b's, we have the three following cases:

Assignment # 7.3c

Case 0: $\alpha_a = \beta_a$, and $\alpha_b = \beta_b$

In which case, using the IH, we have:

$\beta_b \leq \beta_a \rightarrow \alpha_b \leq \alpha_a$

Case 1: $\alpha_b = \beta_b$, and $\alpha_a = \beta_a + 1$

In which case, using the IH, we have:

$\beta_b \leq \beta_a \rightarrow \alpha_b \leq \alpha_a$

Case 2: $\alpha_b = \beta_b + 1$, and $\alpha_a = \beta_a + 1$

In which case, using the IH, we have:

$\beta_b \leq \beta_a \rightarrow \alpha_b \leq \alpha_a$

Case 3: $\alpha_b = \beta_b$, and $\alpha_a = \beta_a + 2$

In which case, using the IH, we have:

$\beta_b \leq \beta_a \rightarrow \alpha_b \leq \alpha_a$