# Key Assignment # 3.1

1. a.) Present a transition diagram for an NFA that recognizes the set of binary strings that start with a 1 and, when interpreted as entering the NFA most to least significant digit; each represents a decimal number that is divisible by either two or five. Thus, 101, 1000, 1111 are in the language, but 111, 1011 and 11011 are not.

   b.) Use the standard conversion technique (subsets of states) to convert the NFA from (a) to an equivalent DFA. Be sure to not include unreachable states.

Construction:

I can do on board, but these are simple variants of ones I already did.

a.) Employ new start state with a non-deterministic transition on an input 1 to two independent DFAs, one of which has two states; the other five. The transitions go to the mod 1 states of each. In the case of divisible by 2, this is a reject state and comes back to an accept only if last digit seen (lsd) is a 0; it must go back to reject when it sees a 1. The other uses the concept that a new lsd, b, when in state k results in a transition to the 2*k+b mod 5 state. Of course that's what the binary one did as well, except mod 2.

b.) Can do as a set of all subset states or just use a parallel construction as we did for union and intersection since it really is the union of two DFAs.

# Practice Problem # 1

Using DFA's (not any equivalent notation) show that the Regular Languages are closed under Min, where $\text{Min}(L) = \{ w \mid w \in L$, but no proper prefix of w is in L$\}$. This means that $w \in \text{Min}(L)$ iff $w \in L$ and for no $y \neq \lambda$ is x in L, where w=xy. Said a third way, w is not an extension of any element in L.

Let $A = (Q, \Sigma, \delta, q_0, F)$ be a DFA such that $L = L(A)$.

Define $A_{MIN} = (Q \cup \{D\}, \Sigma, \delta', q_0, F)$, where D is not in Q.

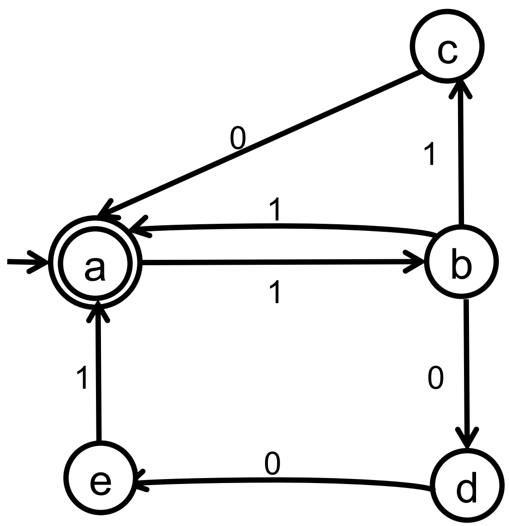
$\delta'$ just changes $\delta$ so that, for each f in F, all its outgoing edges now point to state D, which loops on itself. All other outgoing edges from final states are removed. This means that all extensions of a word in L fail to be recognized. This is just the definition of MIN(L) recast in terms of the behavior of its accepting DFA.

There is a way that breaks out of the DFA and enters the domain of the NFA. One merely removes all edges that start at a final state. One would then need to recast as a DFA, so that's a bit of a cheat, but we will accept it.

A way that also somewhat ignores the constraint of a DFA is to note that DFAs are closed under intersection and complement and so under difference. At this point we can then show that $\text{Min}(L) = L - L\,\Sigma^+$. This is the proof most commonly found on net.

# Variation of Practice Prob. # 2

a.) Present a transition diagram for an NFA for the language associated with the regular expression (1001 + 110 + 11)*. Your NFA must have no more than five states.
b.) Use the standard conversion technique (subsets of states) to convert the NFA from (a) to an equivalent DFA. Be sure to not include unreachable states. Hint: This DFA should have no more than six states.




* Dead State (X)