Assignment # 1.1 Key

1. Prove or disprove the following:
   For non-empty sets A and B, $(A \cup B)=(A \cap B)$ if and only if A=B

   Part 1) Prove if A = B, then $(A \cup B)=(A \cap B)$

   Assume A=B then showing $(A \cup B)=(A \cap B)$ is equivalent to showing $(A \cup A)=(A \cap A)$. Now, any set unioned or intersected with itself is that set. Thus, $(A \cup A) = A$ and $(A \cap A) = A$ and so $(A \cup A)=(A \cap A)$, proving that A = B implies $(A \cup B)=(A \cap B)$. Note: This is true even if both are empty.

   Part 2) Prove if $(A \cup B)=(A \cap B)$, then A = B

   Assume otherwise, then there is some case where $(A \cup B)=(A \cap B)$, but $A \neq B$. This means one set must have an element that is missing from the other. As A's and B's roles are symmetric and each is non-empty, we can choose to say that there is some x in A that is not in B. As x is in A, it is in $(A \cup B)$, but since it is not in B then it is not in $(A \cap B)$, and hence $(A \cup B) \neq (A \cap B)$, but that contradicts our original assumption. Thus, $(A \cup B)=(A \cap B)$ implies A = B.

   Together Parts 1 and 2 show that, for non-empty A and B, $(A \cup B)=(A \cap B)$ if and only if A=B. Note: even here, the non-empty condition is superfluous as $A \neq B$ implies one has an element and we can just choose that one without worrying if the other is empty or not.