Consider the pushdown automaton $\mathcal{A} = (\{ q, f \}, \{ 0, 1, c \}, \{ 0, 1, c, \$ \}, \delta, q, \$, \{ f \})$, where δ defines transitions:

$\delta(q, 0, \$) = \{ (q, \text{PUSH}(1)) \}$
$\delta(q, 1, \$) = \{ (q, \text{PUSH}(0)) \}$
$\delta(q, 0, 0) = \{ (q, \text{PUSH}(1)) \}$
$\delta(q, 0, 1) = \{ (q, \text{PUSH}(1)) \}$
$\delta(q, 1, 0) = \{ (q, \text{PUSH}(0)) \}$
$\delta(q, 1, 1) = \{ (q, \text{PUSH}(0)) \}$
$\delta(q, c, 0) = \{ (p, \text{PUSH}(c)) \}$
$\delta(q, c, 1) = \{ (p, \text{PUSH}(c)) \}$
$\delta(p, \lambda, c) = \{ (p, \text{POP}) \}$
$\delta(p, 0, 0) = \{ (p, \text{POP}) \}$
$\delta(p, 1, 1) = \{ (p, \text{POP}) \}$
$\delta(p, \lambda, \$) = \{ (f, \text{POP}) \}$

This generates the language $\mathcal{L}(\mathcal{A}) = \{ w\ c\ h(w)^R \mid w \in \{0,1\}^+ \}$ **and h(0)=1; h(1)=0**

Write the equivalent grammar using our class's variant of the construction in Hopcroft, Motwani and Ullman.

Hint: the starting non-terminal is: $\langle q, \$, f \rangle$, meaning generate all string that are consumed when we start in **q**, and end up in **f**, having uncovered what's below $\$$.

```
< q, $, f >  → 0 < q, 1, q > < q, $, f >        X
             | 0 < q, 1, p > < p, $, f >
             | 0 < q, 1, f > < f, $, f >        X
             | 1 < q, 0, q> < q, $, f >         X
             | 1 < q, 0, p > < p, $, f >
             | 1 < q, 0, f > < f, $, f >        X
< q, 0, p >  → 0 < q, 1, p > < p, 0, p >
             | 1 < q, 0, p > < p, 0, p >
             | c <p, 0, p >
< q, 1, p >  → 0 < q, 1, p > < p, 1, p >
             | 1 < q, 0, p > < p, 1, p >
             | c <p, 1, p >
< p, 0, p >  → 0
< p, 1, p >  → 1
< p, $, f >  → λ
```

$\langle q, 0, q \rangle$, $\langle q, 1, q \rangle$, $\langle f, \$, f \rangle$ can lead nowhere as states q and f never entered after popping the stack.

Rewrite as

```
S    → 0 T |  1 U
T    → 0 T 1  |  1 U 1 | c 1        // owe you a 1
U    → 0 T 0  |  1 U 0 | c 0        // owe you a 0
```

Consider the pushdown automaton $\mathcal{A}$ **= ({ q, f } , { 0 , 1 , c } , { 0, 1, c }, δ, q, { f }),**
where δ defines transitions:

$\delta(q, 0, \lambda) = \{ (q, PUSH(1)) \}$
$\delta(q, 1, \lambda) = \{ (q, PUSH(0)) \}$
$\delta(q, c, \lambda) = \{ (p, PUSH(c)) \}$
$\delta(p, \lambda, c) = \{ (p, POP) \}$
$\delta(p, 0, 0) = \{ (p, POP), (f, POP) \}$
$\delta(p, 1, 1) = \{ (p, POP), (f, POP) \}$

This generates the language $\mathcal{L}(\mathcal{A}) = \{ w\ c\ h(w)^R \mid w \in \{0,1\}^+ \}$ **and h(0)=1; h(1)=0**
Write the equivalent grammar using book's construction.
Hint: the starting non-terminal is: **Aq,f**, meaning generate all string that are consumed when we start in **q**, and end up in **f**. Note that the stack is empty at start and we allow top of stack to be ignored in transitions.

Aq,f → Aq,q Aq,f	**Useless as becomes Aq,f→Aq,f since can never pop and end in q**
| Aq,p Ap,f	**Useless as can never push starting in p**
| Aq,f Af,f	**Useless as becomes Aq,f→Aq,f (see below)**
| 0 Aq,p 1	
| 1 Aq,p 0	
Aq,p → Aq,q Aq,p	**Useless as becomes Aq,p→Aq,p since can never pop and end in q**
| Aq,p Ap,p	**Useless as becomes Aq,p→Aq,p (see below)**
| 0 Aq,p 1	
| 1 Aq,p 0	
| c Ap,p λ	**This reduces to Aq,p→c**
Ap,p → λ	**No other options as never pushes onto stack**
Af,f → λ	**No other options since never pushes onto stack**

Rewrite as

S → 0 T 1 | 1 T 0
T → 0 T 1 | 1 T 0 | c