

# Example of CNF Conversion

# Starting Grammars

- $L = \{ a^i b^j c^k \mid i=j \text{ or } j=k \}$
- $G = (\{S, A, \langle B=C \rangle, C, \langle A=B \rangle\}, \{a, b\}, R, S)$
- R:
  - $S \rightarrow A \mid C$
  - $A \rightarrow a A \mid \langle B=C \rangle$
  - $\langle B=C \rangle \rightarrow b \langle B=C \rangle c \mid \lambda$
  - $C \rightarrow C c \mid \langle A=B \rangle$
  - $\langle A=B \rangle \rightarrow a \langle A=B \rangle b \mid \lambda$

# Remove Null Rules

- **Nullable = {<B=C>, <A=B>, A, C, S}**
  - $S' \rightarrow S \mid \lambda$  // S' added to V
  - $S \rightarrow A \mid C$
  - $A \rightarrow a A \mid a \mid \langle B=C \rangle$
  - $\langle B=C \rangle \rightarrow b \langle B=C \rangle c \mid b c$
  - $C \rightarrow C c \mid c \mid \langle A=B \rangle$
  - $\langle A=B \rangle \rightarrow a \langle A=B \rangle b \mid ab$

# Remove Unit Rules

- Chains =  $\{[S':S',S,A,C,<A=B>,<B=C>], [S:S,A,C,<A=B>,<B=C>], [A:A,<B=C>],[C:C,<B=C>], [<B=C>:<B=C>], [<A=B>:<A=B>] \}$ 
  - $S' \rightarrow \lambda \mid aA \mid a \mid b<B=C>c \mid bc \mid Cc \mid c \mid a<A=B>b \mid ab$
  - $S \rightarrow aA \mid a \mid b<B=C>c \mid bc \mid Cc \mid c \mid a<A=B>b \mid ab$
  - $A \rightarrow aA \mid a \mid b<B=C>c \mid bc$
  - $<B=C> \rightarrow b<B=C>c \mid bc$
  - $C \rightarrow Cc \mid c \mid a<A=B>b \mid ab$
  - $<A=B> \rightarrow a<A=B>b \mid ab$

# Remove Useless Symbols

- All symbols are productive (lead to terminal string)
- $S$  is useless as it is inaccessible from  $S'$  (new start). All others symbols are accessible from  $S'$ .

# Normalize rhs as CNF

- $s' \rightarrow \lambda \mid \langle a \rangle A \mid a \mid \langle b \rangle \langle \langle B=C \rangle \langle c \rangle \rangle \mid \langle b \rangle \langle c \rangle \mid C \langle c \rangle \mid c \mid \langle a \rangle \langle \langle A=B \rangle \langle b \rangle \rangle \mid \langle a \rangle \langle b \rangle$
- $A \rightarrow \langle a \rangle A \mid a \mid \langle b \rangle \langle \langle B=C \rangle \langle c \rangle \rangle \mid \langle b \rangle \langle c \rangle$
- $\langle B=C \rangle \rightarrow \langle b \rangle \langle \langle B=C \rangle \langle c \rangle \rangle \mid \langle b \rangle \langle c \rangle$
- $C \rightarrow C \langle c \rangle \mid c \mid \langle a \rangle \langle \langle A=B \rangle \langle b \rangle \rangle \mid \langle a \rangle \langle b \rangle$
- $\langle A=B \rangle \rightarrow \langle a \rangle \langle \langle A=B \rangle \langle b \rangle \rangle \mid \langle a \rangle \langle b \rangle$
- $\langle \langle B=C \rangle \langle c \rangle \rangle \rightarrow \langle B=C \rangle \langle c \rangle$
- $\langle \langle A=B \rangle \langle b \rangle \rangle \rightarrow \langle A=B \rangle \langle b \rangle$
- $\langle a \rangle \rightarrow a$
- $\langle b \rangle \rightarrow b$
- $\langle c \rangle \rightarrow c$