COT 4210 Fall 2014 Sample Problems

1. Draw a DFA to recognize the set of strings over **{a,b}***that contain the same number of occurrences of the substring **ab** as of the substring **ba**.

2. Present the transition diagram or table for a DFA that accepts the regular set denoted by the expression **(0+1)* (010 + 11) (0 + 1)***

3. Consider the following assertion:

Let **R** be a regular language, then any set **S**, such that $\mathbf{S} \cup \mathbf{R} = \mathbf{S}$, is also regular.

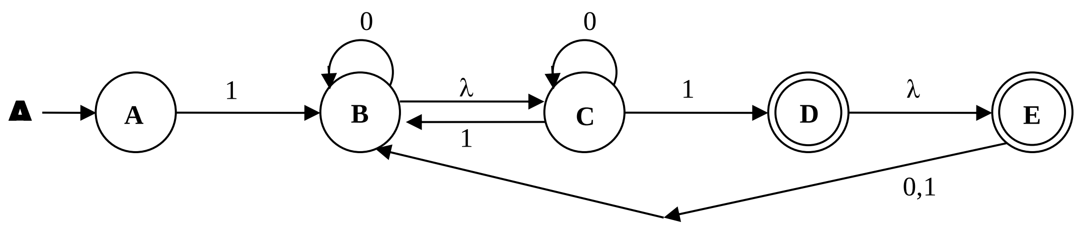
State whether you believe this statement to be True or False by circling your answer.

TRUE **FALSE**

If you believe that this assertion is True, present a convincing argument (not formal proof) to back up your conjecture. If you believe that it is False, present a counterexample using known regular and non-regular languages, **R** and **S**, respectively.

4. Assume that $\mathbf{L_1}$ and $\mathbf{L_1 \cap L_2}$ are both regular languages. Is $\mathbf{L_2}$ necessarily a regular language? If so, prove this, otherwise show that $\mathbf{L_2}$ could either be regular or non-regular.

5. Let **L** be defined as the language accepted by the finite state automaton **A**:

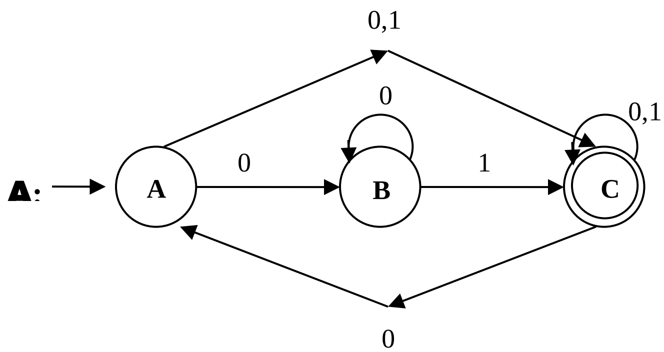


a.) Fill in the following table, showing the λ-closures for each of **A**'s states.

State	**A**	**B**	**C**	**D**	**E**
λ-closure	**{ }**	**{ }**	**{ }**	**{ }**	**{ }**

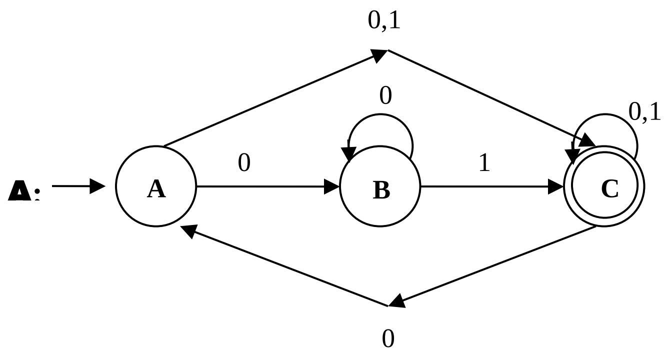
b.) Convert **A** to an equivalent deterministic finite state automaton. Use states like **AC** to denote the subset of states **{A,C}**. Be careful -- λ-closures are important.

6. Let **L** be defined as the language accepted by the finite state automaton **A**:



Change **A** to a GNFA (although I don't see the need for all those ϕ transitions). Now, using the technique of replacing transition letters by regular expressions and then ripping (collapsing) states, develop the regular expression associated with **A** that represents the set (language) **L**.

7. Let **L** be defined as the language accepted by the finite state automaton **A**:



Using the technique of regular equations, develop the regular expression associated with **A** that represents the set (language) **L**.

8. Given a finite state automaton denoted by the transition table shown below, and assuming that **5** and **6** are final states, fill in the equivalent states matrix I have provided. Use this to create an equivalent, minimal state automaton. The start state is **1**.

	a	**b**	**c**
1	**4**	**6**	**2**
2	**5**	**2**	**1**
3	**4**	**5**	**4**
4	**5**	**4**	**3**
5	**4**	**6**	**1**
6	**2**	**6**	**3**

2					
3					
4					
5					
6					
	1	**2**	**3**	**4**	**5**

Don't forget to construct and write down your new, equivalent automaton!!

9. Use the Pumping Lemma to show that the following languages are not regular:

a.) $\mathbf{L = \{ a^n b^m c^t \mid n > m}$ or $\mathbf{n > t,}$ and $\mathbf{n, m, t \geq 0 \}}$

b.) $\mathbf{L = \{ a^n b^m \mid n \leq m,}$ and $\mathbf{n, m \geq 0 \}}$

c. Let **NonPrime** = { $\mathbf{a^q}$ | **q is not a prime** }
This language is not easily shown non-regular using the Pumping Lemma. However, we already saw in class how to prove that **Prime** = { $\mathbf{a^p}$ | **p is not a prime** } is non-regular using the Pumping Lemma. Explain how you could use this latter fact to show **NonPrime** is non-regular.

10. DFAs accept the class of regular languages. Regular expressions denote the class of regular sets. The equivalence of these is seen by a proof that every regular set is a regular language and vice versa. The first part of this, that every regular set is a regular language, can be done by first showing that the basis regular sets (**Ø , { λ } , { a | a** ∈ ∑ }) are each accepted by a DFA.

i.) Demonstrate a DFA for each of the basis regular sets.

Ø

{ λ }

{ a }

Let $\mathbf{L_1}$ be generated by the DFA $\mathbf{A_1 = (Q_1 , \sum , \delta_1 , q_1 , F_1)}$ and $\mathbf{L_2}$ be generated by the DFA $\mathbf{A_2 = (Q_2 , \sum , \delta_2 , q_2 , F_2)}$.

ii.) Present a construction from $\mathbf{A_1}$ **,** $\mathbf{A_2}$ that produces a DFA $\mathbf{A_3}$ for $\mathbf{L_1 \cup L_2}$.
Hint: Cross product.

iii.) What remains to be done to show that every regular set is a regular language? Don't do the proof; just state what two steps still need to be done.