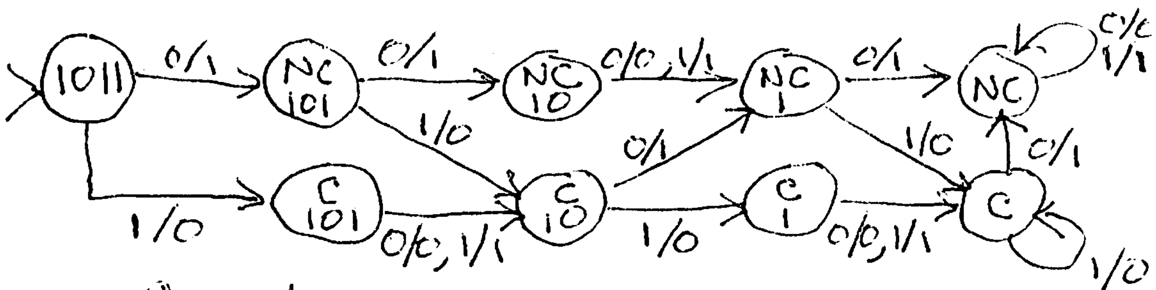
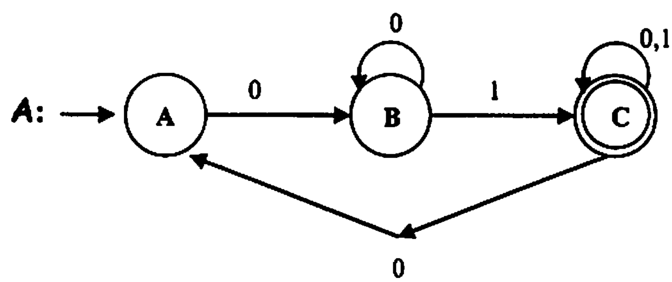
COT 4210 Fall 2014 Midterm 2 Name: KEY
Total Points Available Your Raw Score ________ Grade ________

7 1. Present a Mealy Model finite state machine that reads an input $x \in \{0, 1\}^*$ and produces the binary number that represents the result of adding binary **1011** to **x** (assumes all numbers are positive, including results). Note: The binary number is read from least to most significant bit.
Examples: input **01010** results in output **10101**; input **0011** results in output **1110**



CAN MERGE NC-1 AND C

5 2. Let L_A be defined as the language accepted by the finite state automaton ***A***:



Present a right linear grammar G_A that generates the language L_A.

$G_A = (\{A, B, C\}, \{0, 1\}, R, A)$ where R is:

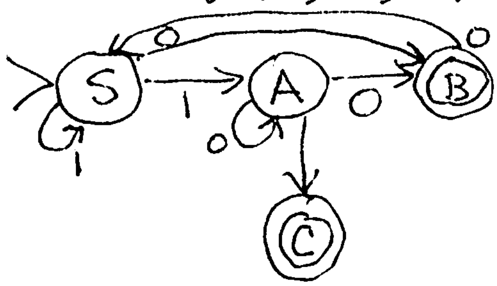
$A \to 0B$
$B \to 0B \mid 1C$
$C \to 0C \mid 1C \mid 0A \mid \lambda$

5 3. Consider the regular grammar **G = ({S, A, B}, {0, 1}, R, S)** where **R** is the set of rules:

S	→	**1 S**	|	**1 A**	|	**0 B**
A	→	**0 B**	|	**0 A**	|	**1**
B	→	**0 S**	|	**λ**		

Present an automaton ***A*** that accepts the language generated by **G**:

$A = (\{S, A, B, C\}, \{0, 1\}, \delta, S, \{B, C\})$



8 **4.** Write a **Context Free Grammar** for the language
L = { a^k b^m c^n | k = n + m, or m = k + n, or n = k + m, k>0, m>0, n>0 }.

$S \Rightarrow aAc \mid aA'bbC'c$
$A \rightarrow aAc \mid aA'b \mid bC'c$
$A' \rightarrow aA'b \mid \lambda$
$C' \rightarrow bC'c \mid \lambda$

5 **5.** Use Myhill-Nerode to show that the language of Problem 4 is not Regular. That language is
L = { a^k b^m c^n | k = n + m, or m = k + n, or n = k + m, k>0, m>0, n>0 }.

CONSIDER CLASSES $[a^i]_{R_L}$ $\forall i>0$
$a^i bc^{i+1} \in L$
HOWEVER,
$a^j bc^{i+1} \notin L$ WHENEVER $j \neq i$
THUS, $[a^i] = [a^j]$ IFF $i = j$
THIS SHOWS AN INFINITE NUMBER OF EQUIVALENCE CLASSES ARE INDUCED BY R_L AND SO L IS NOT REGULAR

8 **6.** Consider the language

L = { $a^n b^{n!}$ | n>0 }.

Use the Pumping Lemma for Context-Free Languages to show that **L** is not context-free.

P.L.: PROVIDES $N>0$

WE: CHOOSE $a^N b^{N!} \in L$

P.L.: SPLITS $a^N b^{N!}$ INTO $uvwxy$, $|vwx| \le N$, $|vx| > 0$ SUCH THAT $\forall i \ge 0$, $uv^iwx^iy \in L$

WE: CHOOSE $i=2$

CASE 1: vwx OVER a'S ONLY. THEN WE ARE INCREASING a'S WHILE LEAVING b'S UNCHANGED, AND $a^{N+c} b^{N!}$, $c>0$ IS NOT IN L.

CAN MERGE CASES 1&3

CASE 2: vwx OVER b'S ONLY. THEN WE ARE INCREASING b'S WHILE LEAVING a'S UNCHANGED, AND $a^N b^{N!+c}$, $c>0$ IS NOT IN L.

CASE 3: vwx CONTAINS SOME a'S & SOME b'S. IT MUST THEN HAVE AT LEAST N+1 a'S AND AT MOST $N!+N-1$ b'S, BUT $(N+1)! = N!(N+1) = N!N + N! \ge N! + N > N! + (N-1)$ AND SO IS NOT IN L.

8 **7.** Present the **CKY** recognition matrix for the string **bbabb** assuming the Chomsky Normal Form grammar, **G = ({S,A,B,C,D }, {a,b}, R, S)**, specified by the rules **R**:

S → AB | BA | BD
A → CS | CD | a
B → DS | b
C → a
D → b

	b	**b**	**a**	**b**	**b**
1	BD	BD	AC	BD	BD
2	S	S	SA	S	
3	B	SB	SA		
4	SB	SB			
5	SB				

4 **8.** Choosing from among **(Y) yes, (N) No**, categorize each of the following closure properties. No proofs are required.

Problem / Language Class (C)	**Regular**	**Context Free**
Closed under union with Context Free languages?	N	Y
Closed under quotient with languages of its own class (C), i.e., L1/L2	Y	N
Closed under difference with languages of its own class (C), i.e., (difference (L1, L2) = L1 – L2)?	Y	N
~~**Closed under union with Context Free languages?**~~		

8 **9.** Prove that any class of languages, C, closed under union, concatenation, intersection with regular languages, homomorphism and substitution (e.g., the Context-Free Languages) is closed under **Erase Middle with Regular Sets (em)**, where **L** $\in$ C, **R** is Regular, **L** and **R** are over the alphabet **Σ**, and **L em R = { xz | x,**$\cancel{y}$**z** $\in \Sigma^+$ **and** $\exists$**y** $\in$ **R, such that xyz** $\in$ **L }**. You may assume substitution **f(a) = {a, a'}**, and homomorphisms **g(a) = a'** and **h(a) = a, h(a') = λ**. Here **a∈Σ** and **a'** is a distinct new character associated with each **a∈Σ**.
You must be very explicit, describing what is produced by each transformation you apply.

$L \text{ em } R = h(f(L) \cap \Sigma^+ \cdot g(R) \cdot \Sigma^+)$

$f(L) = \{ \bar{w} \mid w \in L \}$ WHERE $\bar{w}$ HAS SOME (OR NONE) OF ITS LETTERS PRIMED, $f(L)$ IS A CFL

$g(R) = \{ y' \mid y \in R \}$ WHERE y' HAS ALL ITS LETTERS PRIMED. $g(R)$ IS REGULAR.

$\Sigma^+ \cdot g(R) \cdot \Sigma^+ = \{ xy'z \mid x,z \in \Sigma^+ \;\&\; y \in R \}$
THIS IS REGULAR

$f(L) \cap \Sigma^+ \cdot g(R) \cdot \Sigma^+ = \{ xy'z \mid xyz \in L \;\&\; y \in R \;\&\; x,z \in \Sigma^+ \}$, THIS IS A CFL SINCE $\cap$ OF CFL & REG

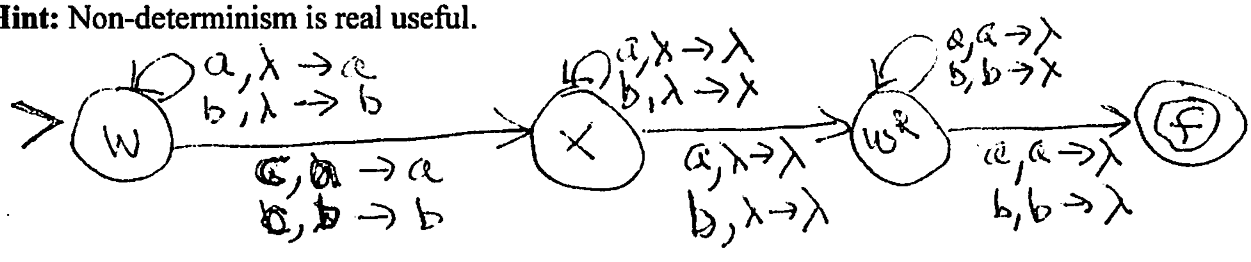
$h(\cdot \;\cdot\; \cdot \;\cdot) = \{ xz \mid x,z \in \Sigma^+, \;\&\; \exists y \in R \ni xyz \in L \}$
$= L \text{ em } R$

8 **10.** Describe a PDA that accepts the following language by empty stack and final state.

{ $wcxw^R$ | w,x ∈ $\{a,b\}^+$ }

Your description must include a state transition diagram or table that indicates start state, initial contents of stack and, of course change to state and stack based on current state, input and stack top.

Hint: Non-determinism is real useful.



START WITH EMPTY STACK

7 **11.** Consider the context-free grammar **G = ({ S, A, B }, { a,b }, R, S)**, where **R** is:

S → SAB | BA
A → AB | a
B → bS | b | λ

a.) Remove all **λ**-rules from **G**, creating an equivalent grammar **G'**. Show all rules.

G':
$S \to SAB \mid SA \mid BA \mid A$
$A \to AB \mid a$
$B \to bS \mid b$

b.) Remove all **unit** rules from **G'**, creating an equivalent grammar **G''**. Show all rules.

G'':
$S \to SAB \mid SA \mid BA \mid AB \mid a$
$A \to AB \mid a$
$B \to bS \mid b$

c.) Convert grammar **G''** to its **Chomsky Normal Form** equivalent, **G'''**. Show all rules.

G''':
$S \to S\langle AB\rangle \mid SA \mid BA \mid AB \mid a$
$A \to AB \mid a$
$B \to \langle b\rangle S \mid b$
$\langle AB\rangle \to AB$
$\langle b\rangle \to b$