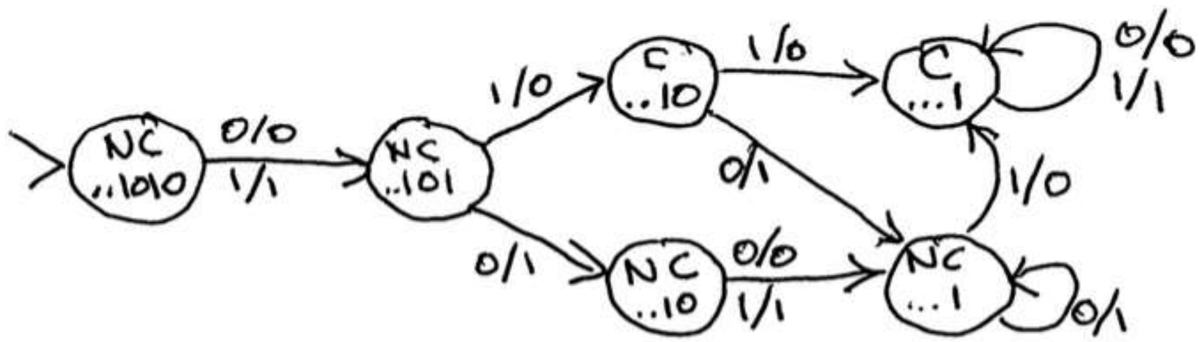
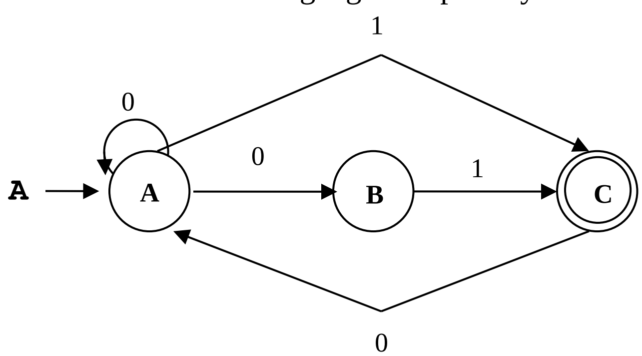
COT 4210 Fall 2014 **Sample Midterm#2** **Name: ________________________**

1. Present a Mealy Model finite state machine that reads an input $\mathbf{x} \in \{0, 1\}^*$ and produces the binary number that represents the result of adding the twos complement representation of decimal **-6**, that is adding binary **1…1010** to **x** (this assumes all numbers are in two's complement notation, including results). Assume that **x** is read starting with its least significant digit.
Examples: **00010 → 11100; 11001 → 10011; 01011 → 00101**



2.

a.) Let **L** be defined as the language accepted by the finite state automaton **A=({A,B,C},{0,1},δ,A,{C})**:



Present a right linear grammar that generates the language **L**.

A → 0A | 0B | 1C

B → 1C

C → 0A | λ

b.) Consider the regular grammar **G = ({S, A, B}, {0, 1}, R, S)**, where **R** is:

S	→	**0 S**	|	**1 A**	|	**0**
A	→	**1 B**	|	**λ**		
B	→	**0 S**				

Present an automaton **A** that accepts the language generated by the **G**:
A=({S,A,B, C},{0,1},δ,S,{A,C}) where δ is:

I'll do in class. It's easy.

3. Assuming you have computed the sets, $\mathbf{R^k_{i,j}}$, for each pair of states, $\mathbf{(q_i,q_j)}$, in some DFA. How is $\mathbf{R^{k+1}_{i,j}}$ calculated, where **k+1** is no greater than the number of states in the associated DFA?

$$R^{k+1}_{i,j} = R^k_{i,j} + R^k_{i,k+1} \left(R^k_{k+1,k+1} \right)^* R^k_{k+1,j}$$

4. Analyze the following language, **L**, proving it is **non**-regular by showing that there are an **infinite** number of equivalence classes formed by the relation $\mathbf{R_L}$ defined by:

$\mathbf{x\ R_L\ y}$ if and only if $[\ \forall \mathbf{z} \in \mathbf{\{a,b,c\}^*},\ \mathbf{xz} \in \mathbf{L}$ exactly when $\mathbf{yz} \in \mathbf{L}\ ]$.

where $\mathbf{L = \{ c^k a^n b^m \mid n < m, k > 0 \}}$.

You don't have to present all equivalence classes, but you must demonstrate a pattern that gives rise to an infinite number of classes, along with evidence that these classes are distinct from one another.

Consider the set of equivalence classes $[ca^ib]$, $i \geq 0$.

Clearly $ca^ib \bullet b^i$ ***is in L, but*** $ca^jb \bullet b^i$ ***is not in L, when j>i.***

Thus, $[ca^ib] \neq [ca^jb]$ ***when j>i leading to an infinite number of equivalence classes induced by*** R_L

Consequently L is not Regular.

5. Write a context-free grammar for the language $\mathbf{L = \{ c^k a^n b^m \mid n < m, k>0 \}}$. Yes, this is the one you just showed is not Regular.

G = ({S,T}, {a,b,c}, R, S) where R is:

S	$\rightarrow$ ***c S***	|	***cT***	
T	$\rightarrow$ ***a T b***	|	***Tb***	| ***b***

6. Which of the following are correct definitions of an ambiguous grammar? Write **T**(true) or **F**(false).

There are two distinct derivations of some string **w** derived by the grammar ***F***

There are two distinct parse trees for some string **w** derived by the grammar ***T***

7. Use the Pumping Lemma for context-free languages to show $\mathbf{L = \{ a^n b^{n^2} \mid n > 0 \}}$ is **not** a CFL.
Be complete and remember to differentiate what you get to do and what the PL gets to do.

P.L: Provide N>0
We: Choose $a^N b^{N^2}$ ***in L***
P.L: Tells us $a^N b^{N^2} = uvwxy$, $|vxy| \leq N$ ***and*** $|vx| > 0$.
We: Choose i=2 and then analyze cases

a) Assume vx contains some a's then uv^2wx2y ***contains at least N+1 a's and at most*** N^2***+N-1 b's. But*** $(N+1)^2 = N^2 + 2N + 1$. ***Thus, the number of b's in*** uv^2wx^2y ***is insufficient in comparison to the number of a's and hence*** uv^2wx^2y ***is not in L.***

b) Assume vx contains no a's the uv^2wx^2y ***contains N a's and at least*** N^2***+1 b's. Thus, the number of b's in*** uv^2wx^2y ***is too many in comparison to the number of a's and so*** uv^2wx^2y ***is not in L.***

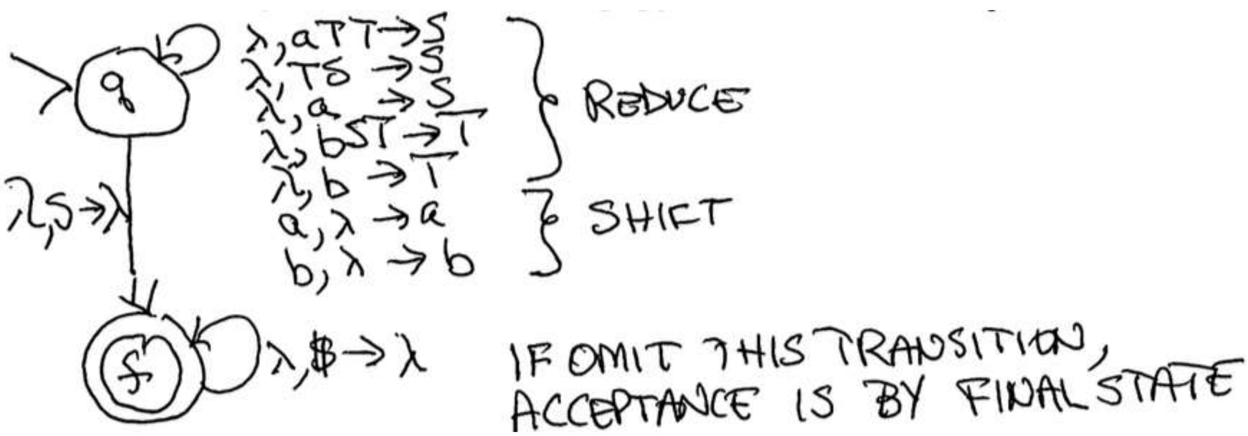
This covers all possibilities and hence L is not Context Free

8. Consider the CFG **G = ({ S, T }, { a, b }, R, S)** where **R** is:

S → a T T | T S | a

T → b S T | b

a.) Present a pushdown automaton that accepts the language generated by this grammar. You may (and are encouraged) to use a transition diagram where transitions have arcs with labels of form **a, α → β** where **a** ∈ Σ∪{λ}, **α, β** ∈ Γ*. Note: I am encouraging you to use extended stack operations.



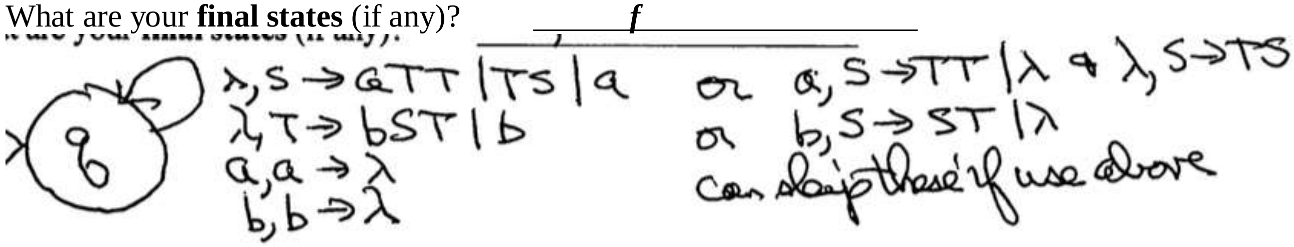
What parsing technique are you using? (Circle one) **top-down** or **bottom-up**

How does your PDA accept? (Circle one) **final state** or **empty stack** or **final state and empty stack**

What is the **initial state**? ***q***

What is the **initial stack content**? ***$***

What are your **final states** (if any)? ***f***



What parsing technique are you using? (Circle one) **top-down** or **bottom-up**

How does your PDA accept? (Circle one) **final state** or **empty stack** or **final state and empty stack**

What is the **initial state**? ***q***

What is the **initial stack content**? ***$***

What are your **final states** (if any)? ***None***

b.) Now, using the notation of **ID**s (Instantaneous Descriptions, **[q, x, z]**), describe how your PDA accepts strings generated by **G**.

[q, w, $] ⇒* [f, λ, λ] if by final state and empty stack (my solution on (a) Bottom-Up)

[q, w, $] ⇒* [f, λ, $] if by final state (I could have done this on (a) Bottom-Up)

[q, w, $] ⇒* [q, λ, λ] if by empty stack (my solution on (a) Top-Down)

9. Consider the context-free grammar **G = ({ S, B, E }, { 0, 1, i, e, s }, R, S)**, where **R** is:

S → i B S E | s
B → 0 | 1
E → λ | e S

a.) Remove all **λ**-rules from **G**, creating an equivalent grammar **G**'. Show all rules, including copied ones.

S → iBS | i B S E | s
B → 0 | 1
E → e S

b.) Convert grammar **G'** to its Chomsky Normal Form equivalent, **G''**. Show all rules, including copied ones from part (a).

S → <I B> S |< i B> < S E> | s
B → 0 | 1
E → <e> S
<i B> → <i> B
<S E>→ S E
<e> → e
<i> → i

10. Let **C** be some class of formal languages that is closed under substitution of members of its own class and under intersection with Regular Languages. Prove that **C** is also closed under **RealWrappers**, where **RealWrappers(L) = { xz | x, z** $\in\Sigma^+$**,** $\exists y\in\Sigma^+$**, w=xyz** $\in$ **L }**. You may assume substitution **f(a) = {a, a'},** and homomorphisms **g(a) = a'** and **h(a) = a, h(a') = λ**. Here **a**$\in\Sigma$ and **a'** is a new character associated with each **a**$\in\Sigma$.

RealWrappers(L) = h(f(L) $\cap\, \Sigma^+ g(\Sigma^+)\, \Sigma^+$ ***)***

11. Fill in the following table with **Y** (yes) or **N** (no) in each cell, depending upon whether or not the class of languages is closed under the stated operation.

	Regular Languages	**Context Free Languages**
Concatenation with Regular	***Y***	***Y***
Quotient with Regular	***Y***	***Y***
Complementation	***Y***	***N***
Superset	***N***	***N***

12. Present the **CKY** recognition matrix for the string **v + (v – v)** assuming the Chomsky Normal Form grammar specified by the grammar
G = ({ E, A, B, T, U, L, R, P, M }, { v, +, –, (,) }, Rules, E), where the **Rules** set is:

E → E A | E B | L U | v
A → P T
B → M T
T → L U | v
U → E R
L → (
R →)
P → +
M → –

	v	**+**	**(**	**v**	**–**	**v**	**)**
1	***E,T***	***P***	***L***	***E,T***	***M***	***E.T***	***R***
2					***B***	***U***	
3				***E***			
4				***U***			
5			***E,T***				
6		***A***					
7	***E***						

Note: Do not be surprised if the above table is sparsely populated.
Hint: Be concerned if it's densely populated.