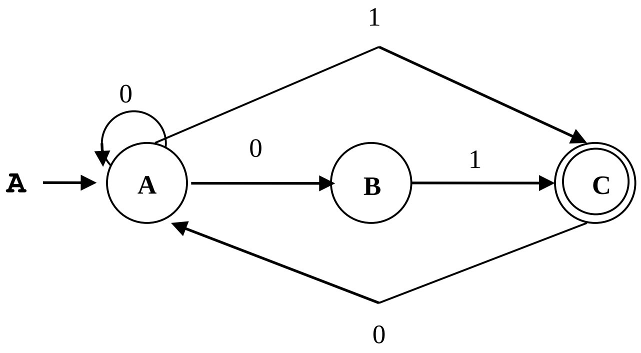
COT 4210 Fall 2014 **Sample Midterm#2** **Name: ________________________**

1. Present a Mealy Model finite state machine that reads an input **x** ∈ **{0, 1}*** and produces the binary number that represents the result of adding the twos complement representation of decimal **-6**, that is adding binary **1…1010** to **x** (this assumes all numbers are in two's complement notation, including results). Assume that **x** is read starting with its least significant digit.
Examples: **00010 → 11100; 11001 → 10011; 01011 → 00101**

2.

a.) Let **L** be defined as the language accepted by the finite state automaton **A=({A,B,C},{0,1},δ,A,{C})**:



Present a right linear grammar that generates the language **L**.

b.) Consider the regular grammar **G = ({S, A, B}, {0, 1}, R, S)**, where **R** is:

S → 0 S | 1 A | 0
A → 1 B | λ
B → 0 S

Present an automaton **A** that accepts the language generated by the **G**:

3. Assuming you have computed the sets, $\mathbf{R^{k}_{i,j}}$, for each pair of states, $\mathbf{(q_i,q_j)}$, in some DFA. How is $\mathbf{R^{k+1}_{i,j}}$ calculated, where **k+1** is no greater than the number of states in the associated DFA?

4. Analyze the following language, **L**, proving it is **non**-regular by showing that there are an **infinite** number of equivalence classes formed by the relation $\mathbf{R_L}$ defined by:

$$\mathbf{x\ R_L\ y} \text{ if and only if } [\ \forall \mathbf{z} \in \mathbf{\{a,b,c\}^*},\ \mathbf{xz} \in \mathbf{L} \text{ exactly when } \mathbf{yz} \in \mathbf{L}\ ].$$

where $\mathbf{L = \{\ c^k\ a^n\ b^m \mid n < m,\ k > 0\ \}}$.

You don't have to present all equivalence classes, but you must demonstrate a pattern that gives rise to an infinite number of classes, along with evidence that these classes are distinct from one another.

5. Write a context-free grammar for the language $\mathbf{L = \{\ c^k\ a^n\ b^m \mid n < m,\ k>0\ \}}$. Yes, this is the one you just showed is not Regular.

6. Which of the following are correct definitions of an ambiguous grammar? Write **T**(true) or **F**(false).

There are two distinct derivations of some string **w** derived by the grammar ________

There are two distinct parse trees for some string **w** derived by the grammar ________

7. Use the Pumping Lemma for context-free languages to show $\mathbf{L = \{ a^n b^{n^2} \mid n > 0 \}}$ is **not** a CFL. *Be complete and remember to differentiate what you get to do and what the PL gets to do.*

8. Consider the CFG **G = ({ S, T }, { a, b }, R, S)** where **R** is:

$\mathbf{S \rightarrow a\,T\,T \mid T\,S \mid a}$
$\mathbf{T \rightarrow b\,S\,T \mid b}$

a.) Present a pushdown automaton that accepts the language generated by this grammar. You may (and are encouraged) to use a transition diagram where transitions have arcs with labels of form $\mathbf{a}, \alpha \rightarrow \beta$ where $\mathbf{a} \in \Sigma \cup \{\lambda\}$, $\alpha, \beta \in \Gamma^*$. Note: I am encouraging you to use extended stack operations.

What parsing technique are you using? (Circle one) **top-down** or **bottom-up**
How does your PDA accept? (Circle one) **final state** or **empty stack** or **final state and empty stack**
What is the **initial state**? __________________________
What is the **initial stack content**? __________________________
What are your **final states** (if any)? __________________________

b.) Now, using the notation of **ID**s (Instantaneous Descriptions, **[q, x, z]**), describe how your PDA accepts strings generated by **G**.

9. Consider the context-free grammar **G = ({ S, B, E }, { 0, 1, i, e, s }, R, S)**, where **R** is:

S → i B S E | s
B → 0 | 1
E → λ | e S

a.) Remove all **λ**-rules from **G**, creating an equivalent grammar **G**'. Show all rules, including copied ones.

b.) Convert grammar **G'** to its Chomsky Normal Form equivalent, **G''**. Show all rules, including copied ones from part (a).

10. Let **C** be some class of formal languages that is closed under substitution of members of its own class and under intersection with Regular Languages. Prove that **C** is also closed under **RealWrappers**, where **RealWrappers(L) = { xz | x, z** $\in\Sigma^+$**,** $\exists y\in\Sigma^+$**, w=xyz** $\in$ **L }**. You may assume substitution **f(a) = {a, a'}**, and homomorphisms **g(a) = a'** and **h(a) = a, h(a') = λ**. Here **a**$\in\Sigma$ and **a'** is a new character associated with each **a**$\in\Sigma$.

11. Fill in the following table with **Y** (yes) or **N** (no) in each cell, depending upon whether or not the class of languages is closed under the stated operation.

	Regular Languages	**Context Free Languages**
Concatenation with Regular		
Quotient with Regular		
Complementation		
Superset		

12. Present the **CKY** recognition matrix for the string **v + (v – v)** assuming the Chomsky Normal Form grammar specified by the grammar
G = ({ E, A, B, T, U, L, R, P, M }, { v, +, –, (,) }, Rules, E), where the **Rules** set is:

E → E A | E B | L U | v
A → P T
B → M T
T → L U | v
U → E R
L → (
R →)
P → +
M → –

	v	**+**	**(**	**v**	**–**	**v**	**)**
1							
2							
3							
4							
5							
6							
7							

Note: Do not be surprised if the above table is sparsely populated.
Hint: Be concerned if it's densely populated.