Sample Assignment # 5.1

For each of the following, prove it is not regular by using the Pumping Lemma or Myhill-Nerode. You must do one of these using the Pumping Lemma and one using Myhill-Nerode.

a. $\{ a^{k!} \mid k>0 \}$ This is set $\{a^1, a^2, a^6, a^{24}, a^{120}, \ldots \}$

b. $\{ a^i b^j c^k \mid i \geq 0, j \geq 0, k \geq 0, j = i + k \}$

Assignment # 5.1 Answer

1a. { $a^{k!}$ | k>0 } using P.L.

1. Assume that L is regular
2. Let N be the positive integer given by the Pumping Lemma
3. Let *s* be a string s = $a^{(N+1)!} \in L$
4. Since *s* $\in$ L and |s| ≥ N, s is split by PL into xyz, where |xy| ≤ N and |y| > 0 and for all i ≥ 0, $xy^iz \in L$
5. We choose i = 2; by PL: xy^2z = xyyz$\in$ L
6. Thus, $a^{(N+1)!+|y|}$ would be $\in$ L. This means that there is a factorial between (N+1)! and (N+1)!+N, but the smallest factorial after (N+1)! Is
   (N+2)! = (N+2) (N+1)! = N(N+1)! + 2(N+1)! > (N+1)! + 2N > (N+1)!+N
7. This is a contradiction, therefore L is not regular ■
8. Note: Using N is dangerous because N could be 1 and 2! Is within N (1) of 1!

1b. { $a^ib^jc^k$ | i≥0, j≥0, k≥0, j = i + k } using P.L.

1. Assume that L is regular
2. Let N be the positive integer given by the Pumping Lemma
3. Let *s* be the string s = $a^Nb^N \in L$
4. Since *s* $\in$ L and |s| ≥ N, s is split by PL into xyz, where |xy| ≤ N and |y| > 0 and for all i ≥ 0, $xy^iz \in L$
5. We choose i = 0; by PL: xz = xz$\in$ L
6. Thus, $a^{N-|y|}b^N$ would be $\in$ L, but it's not since N-|y| + 0 < N. Note: The 0 is because there are 0 c's
7. This is a contradiction, therefore L is not regular ■

Assignment # 5.1 Answer

1a. { $a^{k!}$ | k>0 } using M.N.

We consider the collection of right invariant equivalence classes [$a^{j!-j}$], j ≥ 0.
It's clear that $a^{j!-j}a^j$ is in the language, but $a^{k!-k}a^j$ is not when j < k
This shows that there is a separate equivalence class [$a^{j!-j}$] induced by R_L, for each j ≥ 0.
Thus, the index of R_L is infinite and Myhill-Nerode states that L cannot be Regular. ■

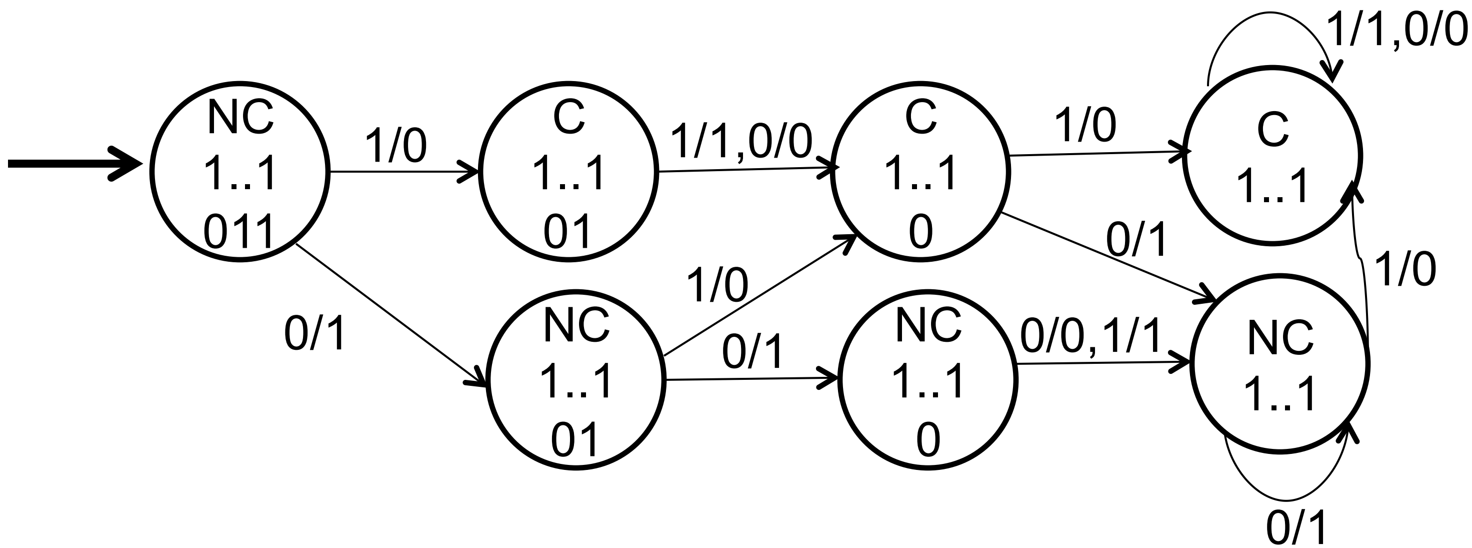
1b. { $a^ib^jc^k$ | i≥0, j≥0, k≥0, j = i + k } using M.N.

We consider the collection of right invariant equivalence classes [a^j], j ≥ 0.
It's clear that a^jb^j is in the language, but a^kb^j is not when j ≠ k
This shows that there is a separate equivalence class [a^j] induced by R_L, for each j ≥ 0.
Thus, the index of R_L is infinite and Myhill-Nerode states that L cannot be Regular. ■

Assignment # 5.2

Write a Mealy finite state machine that produces the 2's complement result of subtracting 101 from a binary input stream (assuming at least 3 bits of input)

Answer



Assignment # 5.3

Write a regular (right linear) grammar that generates the set of strings denoted by the regular expression (0+11)* (101 (00 + 1)*)*

G = ({<0+11>,<1_1>,<1_01>,<10_1>,<00+11>,<0_0>}, {0,1}, R, <0+11>)

<0+11> → 0<0+11> | 1<1_1> | 1<1_01> | λ

<1_1> → 1<0+11>

<1_01> → 0<10_1>

<10_1> → 1<00+1>

<00+1> → 0<0_0> | 1<00+1> | λ | 1<1_01>

<0_0> → 0<00+1>

Or

G = ({S,T,U,V,W,X}, {0,1}, R, S)

S → 0S | 1T | 1U | λ

T → 1S

U → 0V

V → 1W

W → 0X | 1W | λ | 1U

X → 0W