Sample Assignment # 2.2 (I have no sample for 2.1)

Present a language L over $\Sigma = \{a\}$ where $L^3 = L^4$ but $L \neq L^2$ and $L^2 \neq L^3$
Note: $L^k = \{ x_1x_2\ldots x_k \mid x_1,x_2,\ldots,x_k \in L \}$

Proof:

Consider $L = \{a\}^* - \{aa, aaa\}$

$L^2 = \{a\}^* - \{aaa\}$ since the presence of the empty string in $\{a\}^*$ means all strings in L are in L^2. Additionally, $aa = a \circ a$ and so aa is in L^2 but aaa is not since it cannot be formed from any pair of members in L

$L^3 = \{a\}^*$ since the presence of the empty string in $\{a\}^*$ means all strings in L^2 are in L^3

Additionally, $aaa = aa \circ a$ and so aaa is in L^3

$L^3 = L^4$ since L^3 is already $\{a\}^*$ and so nothing new can be created and the presence of the empty string in $\{a\}^*$ means all in L^3 are in L^4