Assignment # 5.1

For each of the following, either show it is regular by producing a regular grammar that generates the language or prove it is not regular by using the Pumping Lemma or Myhill-Nerode. You must do at least two of these using the Pumping Lemma and at least two using Myhill-Nerode.

a. $\{ a^{Fib(k)} \mid k>0 \}$ This is $\{a^1, a^1, a^2, a^3, a^5, a^8, a^{13}, a^{21}, \dots \}$

b. $\{ a^i b^j c^k \mid i \geq 0, j \geq 0, k \geq 0, k = \min(i,j) \}$

c. $\{ a^i b^j c^k \mid i \geq 0, j \geq 0, k \geq 0, j = i * k \}$

d. $\{ a^i b^j c^k \mid i \geq 0, j \geq 0, k \geq 0, \text{if } i=1 \text{ then } j>k \}$

e. $\{ w \mid w \in \{a, b\}^* \text{ and } w = w^R \}$ this is the set of palindromes. It contains strings like aa, abba, abaaba

Assignment # 5.1 Answer

1a, $\{ a^{Fib(k)} \mid k>0 \}$ using P.L.

1. Assume that L is regular
2. Let N be the positive integer given by the Pumping Lemma
3. Let *s* be a string $s = a^{Fib(N+3)} \in L$
4. Since $s \in L$ and $|s| \geq N$ (Fib(N+3)>N in all cases; actually Fib(N+2)>N as well), s is split by PL into xyz, where $|xy| \leq N$ and $|y| > 0$ and for all $i \geq 0$, $xy^iz \in L$
5. We choose i = 2; by PL: $xy^2z = xyyz \in L$
6. Thus, $a^{Fib(N+3)+|y|}$ would be $\in L$. This means that there is Fibonacci number between Fib(N+3) and Fib(N+3)+N, but the smallest Fibonacci greater than Fib(N+3) is Fib(N+3)+Fib(N+2) and Fib(N+2)>N This is a contradiction, therefore L is not regular ■
7. Note: Using values less than N+3 could be dangerous because N could be 1 and both Fib(2) and Fib(3) are within N (1) of Fib(1).

1b, $\{ a^ib^jc^k \mid i\geq 0, j\geq 0, k\geq 0, k = \min(i,j) \}$ using P.L.

1. Assume that L is regular
2. Let N be the positive integer given by the Pumping Lemma
3. Let *s* be the string $s = a^Nb^{N+1}c^N \in L$
4. Since $s \in L$ and $|s| \geq N$, s is split by PL into xyz, where $|xy| \leq N$ and $|y| > 0$ and for all $i \geq 0$, $xy^iz \in L$
5. We choose i = 0; by PL: $xz = xz \in L$
6. Thus, $a^{N-|y|}b^{N+1}c^N$ would be $\in L$, but it's not since $N-|y| < N < N+1$
7. This is a contradiction, therefore L is not regular ■

Assignment # 5.1 Answer

1c, { $a^i b^j c^k$ | i≥0, j≥0, k≥0, j = i * k } using P.L.

1. Assume that L is regular
2. Let N be the positive integer given by the Pumping Lemma
3. Let *s* be the string s = $a^N b^N c \in L$
4. Since *s* ∈ L and |s| ≥ N, s is split by PL into xyz, where |xy| ≤ N and |y| > 0 and for all i ≥ 0, $xy^iz \in L$
5. We choose i = 0; by PL: xz = xz∈ L
6. Thus, $a^{N-|y|} b^N c$ would be ∈ L, but it's not since (N-|y|) * 1 = N-|y| < N.
7. This is a contradiction, therefore L is not regular ■

1d, { $a^i b^j c^k$ | i≥0, j≥0, k≥0, if i=1 then j>k } using P.L.

1. This is one I would personally never choose to solve using the Pumping Lemma. Assume that L is regular. The problem is that, if I start with a single **a**, the PL might break my string up so y=**a** and now erasing it or increasing the number of **a**'s is meaningless because there is no constraint on **b**'s and **c**'s when there is no **a** or many **a**'s. Alternately, if I start with lots of **a**'s, I have to guarantee I can reduce these to precisely one **a**. So what's a person to do, other than apply Myhill-Nerode? Well, there is a way because L is regular iff L^R is regular, so we can prove L^R is non-regular.
2. Let N be the positive integer given by the Pumping Lemma as regards L^R
3. Let *s* be the string s = $c^N b^{N+1} a \in L$
4. Since *s* ∈ L and |s| ≥ N, s is split by PL into xyz, where |xy| ≤ N and |y| > 0 and for all i ≥ 0, $xy^iz \in L$
5. We choose i = 2; by PL: xz = xz∈ L
6. Thus, $c^{N+|y|} b^N a$ would be ∈ L, but it's not since N+|y| ≥ N+1.
7. This is a contradiction, therefore L is not regular ■

Assignment # 5.1 Answer

1e, { w | w $\in$ {a, b}* and w = w^R } using P.L.

1. Assume that L is regular
2. Let N be the positive integer given by the Pumping Lemma
3. Let *s* be the string s = $a^N b a^N \in L$
4. Since *s* $\in$ L and |s| ≥ N, s is split by PL into xyz, where |xy| ≤ N and |y| > 0 and for all i ≥ 0, $xy^iz \in L$
5. We choose i = 0; by PL: xz = xz$\in$ L
6. Thus, $a^{N-|y|}ba^N$ would be $\in$ L, but it's not since N-|y| ≠ N.
7. This is a contradiction, therefore L is not regular ■

Assignment # 5.1 Answer

1a, { $a^{Fib(k)}$ | k>0 } using M.N.
We consider the collection of right invariant equivalence classes [$a^{Fib(j)}$], j > 2.
It's clear that $a^{Fib(j)}a^{Fib(j+1)}$ is in the language, but $a^{Fib(k)}a^{Fib(j+1)}$ is not when k>2 and k≠j and k≠j+2
This shows that there is a separate equivalence class [$a^{Fib(j)}$] induced by R_L, for each j > 2.
Thus, the index of R_L is infinite and Myhill-Nerode states that L cannot be Regular. ■

1b, { $a^ib^jc^k$ | i≥0, j≥0, k≥0, k = min(i,j) } using M.N.
We consider the collection of right invariant equivalence classes [a^j], j ≥ 0.
It's clear that a^jc^j is in the language, but a^kc^j is not when j ≠ k
This shows that there is a separate equivalence class [a^j] induced by R_L, for each j ≥ 0.
Thus, the index of R_L is infinite and Myhill-Nerode states that L cannot be Regular. ■

1c, { $a^ib^jc^k$ | i≥0, j≥0, k≥0, j = i * k } using M.N.
We consider the collection of right invariant equivalence classes [a^j], j ≥ 0.
It's clear that a^jb^jc is in the language, but a^kb^jc is not when j ≠ k
This shows that there is a separate equivalence class [a^j] induced by R_L, for each j ≥ 0.
Thus, the index of R_L is infinite and Myhill-Nerode states that L cannot be Regular. ■

Assignment # 5.1 Answer

1d, { $a^i b^j c^k$ | i≥0, j≥0, k≥0, if i=1 then j>k } using M.N.

We consider the collection of right invariant equivalence classes [ab^j], j ≥ 1.

It's clear that ab^jc^{j-1} is in the language, but ab^mc^{j-1} is not when m<j

This shows that there is a separate equivalence class [a^j] induced by R_L, for each j ≥ 1.

Thus, the index of R_L is infinite and Myhill-Nerode states that L cannot be Regular. ■

1e, { w | w $\in$ {a, b}* and w = w^R } using M.N.

We consider the collection of right invariant equivalence classes [a^jb], j ≥ 0.

It's clear that a^jba^j is in the language, but a^kba^j is not when j ≠ k

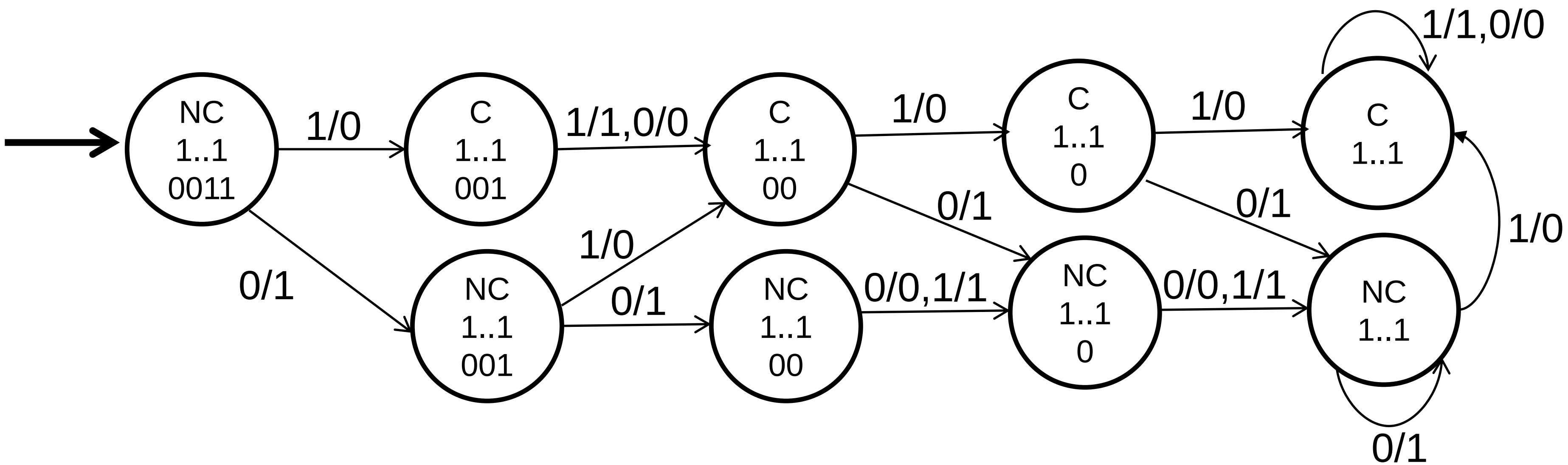
This shows that there is a separate equivalence class [a^j] induced by R_L, for each j ≥ 0.

Thus, the index of R_L is infinite and Myhill-Nerode states that L cannot be Regular. ■

Assignment # 5.2

Write a Mealy finite state machine that produces the 2's complement result of subtracting 1101 from a binary input stream (assuming at least 3 bits of input)

Answer



Assignment # 5.3

Write a regular (right linear) grammar that generates the set of strings denoted by the regular expression $((10)^+ (011 + 1)^+)^* (0+101)^*$

$G = (\{S,T,U\}, \{0,1\}, R, S)$

R:

$S \rightarrow 10S \mid 10T \mid 0U \mid 101U \mid \lambda$

$T \rightarrow 011T \mid 1T \mid 011S \mid 1S$

$U \rightarrow 0U \mid 101U \mid \lambda$