Assignment # 2.1 Key

Let L be a language over {a,b} where every string is of even length and is of the form WX, where |W|=|X| but W≠X. Design and present an algorithm that recognized strings in L using no unbounded amount of storage (no stacks, no queues). This means that any memory required must be of a fixed size independent of the length of an input string. Note: You cannot play the game of using unbounded recursion, as each call consumes stack space.

Proof:

You can attack this deterministically or non-deterministically. I will do deterministically. Consider any string z=WX, |W|=|X| but W≠X. Such a string need only have one transcription error when copying W as X to be in L. check fits the bill.

```
int check(const char *z){ int p = -1; int odd = 0; int mid;
    while (z[++p]) { odd = 1-odd; }
    if ( odd ) return 0;
    mid = p/2;
    for (p=0; p<mid; p++) if (z[p] != z[mid+p]) return 1;
    return 0;
}
```

Assignment # 2.2 Key

Present a language L over $\Sigma = \{a\}$ where $L^4 = L^5$ but $L \neq L^2$, $L^2 \neq L^3$ and $L^3 \neq L^{34}$
Note: $L^k = \{ x_1x_2\ldots x_k \mid x_1,x_2,\ldots,x_k \in L \}$. This is basically a giveaway, since I showed exactly how to do it.

Proof:

Consider $L = \{a\}^* - \{aa, aaa, aaaa\}$

$L^2 = \{a\}^* - \{aaa, aaaa\}$ since the presence of the empty string in $\{a\}^*$ means all strings in L are in L^2. Additionally, $aa = a \circ a$ and so aa is in L^2 but aaa and aaaa are not since they cannot be formed from any pair of members in L

$L^3 = \{a\}^* - \{aaaa\}$ since the presence of the empty string in $\{a\}^*$ means all strings in L are in L^3. Additionally, $aaa = aa \circ a$ and so aaa is in L^3 but aaaa is not since it cannot be formed from any triple of members in L

$L^4 = \{a\}^*$ since the presence of the empty string in $\{a\}^*$ means all strings in L^3 are in L^3. Additionally, $aaaa = aaa \circ a$ and so aaaa is in L^4

$L^4 = L^5$ since L^4 is already $\{a\}^*$ and so nothing new can be created and the presence of the empty string in $\{a\}^*$ means all in L^4 are in L^5