COT 3100 Final Exam - Part A (Relations, Functions) - 25 pts (12/3/2024) Solutions

1) (5 pts) How many anti-symmetric relations over the set A = {1, 2, 3, 4, 5, 6, 7, 8} contain the ordered pairs (1, 1), (2, 3), (6, 4), (7, 7), and (8, 3)? Please leave your answer as a product of terms, each of which is written in exponential form.

We must include (1, 1) and (7, 7). We can either include or not include (2, 2), (3, 3), (4, 4), (5, 5), (6, 6) and (8, 8). We can make these selections in 2^6 ways for these 6 ordered pairs.

We are not allowed to include (3, 2), (4, 6) and (3, 8), because if we did, the relation would not be anti-symmetric.

Consider the other 25 ordered pairs of pairs of the form ((a, b), (b, a)) where a ≠ b. (To figure out that there are 25 of these, note that we start with 8 x 8 = 64 possible ordered pairs in the relation. Then we subtract out the 8 of the form (a, a) to get 64 – 8 = 56. Next, we subtract out the pairs (2, 3), (3, 2), (6, 4), (4, 6), (8, 3) and (3, 8) because we've already accounted for the membership of these ordered pairs. This leaves us with 56 – 6 = 50 ordered pairs, which we put in 25 groups, each with 2 ordered pairs. For each of these 25 groups, we can choose one of three things: put nothing into our relation, put (a, b) in our relation or put (b, a) in our relation. (We're not allowed to put both (a, b) and (b, a) in the relation and maintain anti-symmetry. Thus, we can make our choices of inclusion on these 50 ordered pairs in 3^{25} ways.

Since any subset from the first part can be paired with any subset from the second part, we multiply the number of choices from both parts to get a final answer of **$2^6 3^{25}$.**

$2^6 3^{25}$ (Grading: 2 pts first term, 2 pts second term, 1 pt mult both terms)

2) (6 pts) Let R = { (a, b) | $\exists c \in Z^+$ $such\ that\ ab = c^2$} be a relation defined over the positive integers. Prove that R is transitive. (You may use the fact that if a rational number squared is an integer, then that rational number itself is also an integer.)

We must show that if $(a, b) \in R \wedge (b, c) \in R$, then $(a, c) \in R$. **(Grading: 1 pt)**

We use direct proof. Let $(a, b) \in R \wedge (b, c) \in R$. This means that there are integers d and e such that: $ab = d^2$ and $bc = e^2$. **(Grading: 1 pt)** Multiply these equations together to get:

$(ab)(bc) = d^2e^2$
$(ac)b^2 = d^2e^2$
$ac = \frac{(de)^2}{b^2} = (\frac{de}{b})^2$, **(Grading: 2 pts)**

At this point, note that the RHS is a rational number squared. But because a and c are positive integers, it's also an integer. Using the fact given, this means that the rational number that is being squared is in fact an integer. We can conclude that $\frac{de}{b} \in Z$, thus by definition of R, $(a, c) \in R$. **(Grading: 2 pts for proper communication)**

3) (8 pts) Let $f: A \to B$ and $g: B \to C$ be functions where f is a surjective function and g is an injective function.

(a) Prove or disprove: the composition function, $g^\circ f$ is a surjective function.
(b) Prove or disprove: the composition function, $g^\circ f$ is an injective function.

For each part, clearly state whether the assertion is true or not, followed by a proper justification.

(a) This claim is false. It's possible that $g^\circ f$ is not surjective.

Let A = {1, 2, 3}, B = {4, 5} and C = {6, 7, 8}.
Let *f* = {(1,4), (2,4), (3, 5)} and *g* = {(4, 6), (5, 8)}.

For this particular example, $g^\circ f$ = {(1,6), (2, 6), (3, 8)}. This function isn't surjective because there is no value, a, in the set A such that $g^\circ f(a) = 7$, but $7 \in C$.

(b) This claim is also false. In the previously stated counter-example, the function $g^\circ f$ is not injective because $g^\circ f(1) = g^\circ f(2) = 6$.

Grading, for both parts:

1 pt for clearly stating that the claim is false (for both parts), if this isn't clearly stated for a part, automatic 0/4 for the part.

2 pts for clearly specifying a valid counter-example with A, B, C, f, g and g(f(x)) fully written out. (Give 1 pt if the counter-example is invalid or incomplete)

1 pt for showing that the counter-example is just that.

4) (6 pts) Let r, s and t be roots of the cubic polynomial $f(x) = 3x^3 - 5x^2 + 7x - 9$. What is the cubic equation, with **leading coefficient 9,** with the roots $\frac{1}{r}$, $\frac{1}{s}$, and $\frac{1}{t}$?

Using the given information, we can set up the following equations that r, s and t satisfy:

$$r + s + t = \frac{5}{3},\ rs + rt + st = \frac{7}{3},\ rst = \frac{9}{3}$$

In order to get a cubic with the roots $\frac{1}{r}$, $\frac{1}{s}$, and $\frac{1}{t}$, we need to find the values of the following expressions:

$\frac{1}{r} + \frac{1}{s} + \frac{1}{t}$, $\frac{1}{rs} + \frac{1}{rt} + \frac{1}{st}$, and $\frac{1}{rst}$.

Let's solve for these:

$$\frac{1}{r} + \frac{1}{s} + \frac{1}{t} = \frac{st}{rst} + \frac{rt}{rst} + \frac{rs}{rst} = \frac{rs+rt+st}{rst} = \frac{\frac{7}{3}}{\frac{9}{3}} = \frac{7}{9}$$

$$\frac{1}{rs} + \frac{1}{rt} + \frac{1}{st} = \frac{t}{rst} + \frac{s}{rst} + \frac{r}{rst} = \frac{r+s+t}{rst} = \frac{\frac{5}{3}}{\frac{9}{3}} = \frac{5}{9}$$

$$\frac{1}{rst} = \frac{1}{\frac{9}{3}} = \frac{3}{9}$$

It follows that a cubic with these roots is $g(x) = x^3 - \frac{7}{9}x^2 + \frac{5}{9}x - \frac{3}{9}$. We can multiply this cubic through by any constant and not change its roots. Since the question requires a leading coefficient of 9, we multiply g(x) above by 9 to get the final answer: $\mathbf{9x^3 - 7x^2 + 5x - 3.}$

$\underline{\mathbf{9x^3 - 7x^2 + 5x - 3}}$

Note: the pattern seen here that all the coefficients are in "reverse" order, following the sign switching rule is true in general. Just plug in $\frac{1}{x}$ for *x* in any arbitrary polynomial where we know 0 isn't a root (constant coefficient that is non-zero) and multiply through by x^n, where *n* is the degree of the polynomial and then simplify!

Grading: **1 pt writing down equations with r, s and t**
2 pts solving for 1/r + 1/s + 1/t
1 pt solving for 1/rs + 1/rt + 1/st
1 pt solving for 1/(rst)
1 pt mult by 9

<u>COT 3100 Final Exam - Part B - 100 pts (12/3/2024) Solution</u>

1) (8 pts) Please complete the truth table below.

p	q	r	$p \to q$	$\bar{q} \to r$	$(p \to q) \to (\bar{q} \to r)$
F	F	F	T	F	F
F	F	T	T	T	T
F	T	F	T	T	T
F	T	T	T	T	T
T	F	F	F	F	T
T	F	T	F	T	T
T	T	F	T	T	T
T	T	T	T	T	T

Grading: 1 pt per row, full row has to be correct to get the point.

2) (10 pts) Let A, B and C be finite sets of integers. Prove the following assertion **<u>via direct proof:</u>**

If $C \subseteq A \cap B$, then $A - B \subseteq A - C$.

To prove this directly, take an arbitrarily chosen element $x \in A - B$. **Grading: 1 pt**
For this arbitrarily chosen element we must prove that $x \in A - C$. **Grading: 1 pt**

By definition of set difference, we have $x \in A \land x \notin B$. **Grading: 2 pts**

By definition of set intersection, if $x \notin B$, then it follows that $x \notin A \cap B$. **Grading: 2 pts**

Finally, since $C \subseteq A \cap B$, a direct consequence of $x \notin A \cap B$ is that $x \notin C$, because if $x \in C$, then it would follow that $x \in A \cap B$, but the latter is false, which means the former must be as well. (Alternatively, we can take the contrapositive of the direct definition of subset and apply it directly.) **Grading: 2 pts**

Now, since we have $x \in A \land x \notin C$, we can conclude via definition of set difference, $x \in A - C$, as desired. **Grading: 2 pts**

3) (10 pts) Find all ordered pairs of integers, (x, y), that satisfy the equation 275x + 64y = 1. Put a box around your final answer.

First, run the Euclidean Algorithm with 275 and 64:

275 = 4 x 64 + 19
64 = 3 x 19 + 7
19 = 2 x 7 + 5
7 = 1 x 5 + 2
5 = 2 x 2 + 1

Now, run the Extended Euclidean Algorithm:

5 – 2 x 2 = 1
5 – 2(7 – 5) = 1
5 – 2 x 7 + 2 x 5 = 1
3 x 5 – 2 x 7 = 1
3(19 – 2 x 7) – 2 x 7 = 1
3 x 19 – 6 x 7 – 2 x 7 = 1
3 x 19 – 8 x 7 = 1
3 x 19 – 8(64 – 3 x 19) = 1
3 x 19 – 8 x 64 + 24 x 19 = 1
27 x 19 – 8 x 64 = 1
27(275 – 4 x 64) – 8 x 64 = 1
27 x 275 – 108 x 64 – 8 x 64 = 1
27 x 275 – 116 x 64 = 1

Thus, one integer solution to the given equation is x = 27 and y = -116.

Recall that we can obtain all other integer solutions by adding and subtracting offsets from any base solution. The offsets are the numbers themselves divided by their gcd. Luckily, since gcd(275, 64) = 1, no division is necessary and our offsets are +64 and -275, respectively. It follows that all solutions can be represented by the following set:

$\{(\boldsymbol{x}, \boldsymbol{y}) | \boldsymbol{x} = \mathbf{27} + \mathbf{64}\boldsymbol{c}, \boldsymbol{y} = -\mathbf{116} - \mathbf{275}\boldsymbol{c}, \boldsymbol{c} \in \boldsymbol{Z}\}$ **}**

Grading: **2 pts Euclidean**
5 pts Extended Euclidean
1 pt extract one solution
1 pt offsets in right places
1 pt correct signs by offsets (just have to be opposite)

4) (10 pts) **Prove via induction on n**, that for all non-negative integers n, $7^n - (-1)^n 3^{2n}$ is divisible by 16.

Base case: $n = 0$, $7^0 - (-1)^0 3^{2(0)} = 1 - 1 = 0$, $16 \mid 0$ because $0 = 0 \times 16$. Thus the base case holds.

Inductive hypothesis: Assume for an arbitrarily chosen non-negative integer $n = k$ that $16 \mid (7^k - (-1)^k 3^{2k})$. Specifically, there exists an integer c such that $16c = 7^k - (-1)^k 3^{2k}$, which means that $(-1)^k 3^{2k} = 7^k - 16c$.

Inductive step: Prove for $n = k+1$ that $16 \mid\mid (7^{k+1} - (-1)^{k+1} 3^{2(k+1)})$.

$$
\begin{aligned}
7^{k+1} - (-1)^{k+1} 3^{2(k+1)}) &= 7(7^k) - (-1)(-1)^k 3^2 3^{2k} \\
&= 7(7^k) + 9(-1)^k 3^{2k} \\
&= 7(7^k) + 9(7^k - 16c), \text{ via the inductive hypothesis} \\
&= 7(7^k) + 9(7^k) - 9(16c) \\
&= 16(7^k) - 9(16c) \\
&= 16(7^k - 9c)
\end{aligned}
$$

Since k is a non-negative integer and c is an integer, it follows that $7^k - 9c$ is an integer. Thus, we can conclude that $16 \mid 7^{k+1} - (-1)^{k+1} 3^{2(k+1)})$, proving the inductive step.

We can conclude that for all non-negative integers n, $16 \mid (7^n - (-1)^n 3^{2n})$.

Grading:
1 pt – base case
1 pt – stating IH
1 pt – stating IS
2 pts – factor out 7, -1, 3^2 correctly
1 pt – get to state where ready to plug in IH
2 pts – plug in IH and state it
2 pts – completing the problem after plugging in IH

5) (10 pts) **Prove via induction on n,** that for all integers n ≥ 2, $\sum_{i=2}^{n} \frac{i}{i^2-1} = \frac{1}{2}(H_{n-1} + H_{n+1}) - \frac{3}{4}$, where H_n represents the n^{th} Harmonic number. (For reference, $H_n = \sum_{i=1}^{n} \frac{1}{i}$.) As a courtesy, completing this problem requires a partial fraction decomposition, which some students might not have learned. To that end, please feel free to use the fact that for all positive real numbers a,

$$\frac{a+1}{a^2+2a} = \frac{1}{2a} + \frac{1}{2(a+2)}$$

Base case: n = 2:

LHS = $\sum_{i=2}^{2} \frac{i}{i^2-1} = \frac{2}{2^2-1} = \frac{2}{3}$

RHS = $\frac{1}{2}(H_{2-1} + H_{2+1}) - \frac{3}{4} = \frac{1}{2}(H_1 + H_3) - \frac{3}{4} = \frac{1}{2}\left(1 + 1 + \frac{1}{2} + \frac{1}{3}\right) - \frac{3}{4} = \frac{1}{2} \times \frac{17}{6} - \frac{9}{12} = \frac{8}{12} = \frac{2}{3}$

Thus, the given statement is true for n = 2.

Inductive Hypothesis: Assume for an arbitrarily chosen positive integer n = k, with k ≥ 2 that

$$\sum_{i=2}^{k} \frac{i}{i^2-1} = \frac{1}{2}(H_{k-1} + H_{k+1}) - \frac{3}{4}$$

Inductive Step: Prove for n = k+1 that $\sum_{i=2}^{k+1} \frac{i}{i^2-1} = \frac{1}{2}(H_k + H_{k+2}) - \frac{3}{4}$

$\sum_{i=2}^{k+1} \frac{i}{i^2-1} = \left(\sum_{i=2}^{k} \frac{i}{i^2-1}\right) + \frac{k+1}{(k+1)^2-1}$

$= \frac{1}{2}(H_{k-1} + H_{k+1}) - \frac{3}{4} + \frac{k+1}{k^2+2k+1-1}$, via the Inductive Hypothesis

$= \frac{1}{2}(H_{k-1} + H_{k+1}) - \frac{3}{4} + \frac{k+1}{k^2+2k}$

$= \frac{1}{2}(H_{k-1} + H_{k+1}) - \frac{3}{4} + \frac{1}{2k} + \frac{1}{2(k+2)}$

$= \frac{1}{2}\left(H_{k-1} + \frac{1}{k} + H_{k+1} + \frac{1}{k+2}\right) - \frac{3}{4}$, factoring out ½ from last two terms

$= \frac{1}{2}(H_k + H_{k+2}) - \frac{3}{4}$, via definition of the Harmonic numbers.

This completes the proof of the inductive step. We can conclude that for all positive integers n≥2 that $\sum_{i=2}^{n} \frac{i}{i^2-1} = \frac{1}{2}(H_{n-1} + H_{n+1}) - \frac{3}{4}$.

Grading: **base case = 1 pt, IH = 1 pt, IS = 1 pt**
sum split = 1 pt, IS sub = 2 pts
algebra on split term to get to (k+1)/(k²+2k) – 1 pt
partial fraction split = 1 pt
problem finish via harmonic recursive def – 2 pts

6) (10 pts) Cassie reads her digital clock and notices that the digits from left to right are in strictly increasing order. (Examples of what she could have read are 2:37 and 12:59.) Assuming that Cassie read her clock in between 12:00 PM and 11:59 PM, inclusive, at how many different times/minutes could she have read her clock? Another way to ask the question is: for how many different valid settings of the 3 or 4 digits on a digital clock are all the digits in strictly increasing order? Note: This problem requires some casework. There's no elegant solution that I know of. I'll be impressed if someone finds one!

The casework is due to the first digit in the minute setting. This digit is limited to the range [0, 5] because there are only 60 minutes in an hour. (Otherwise a clean combination solution similar to a past recitation problem would work.)

Since the last (units/ones digit) hour digit in any valid response must be at least 1 ("10:" isn't increasing), this means our actual range we need to consider for the middle digit is [2, 5]. So we can split up our counting into four cases:

Leftmost minute digit = 2, Hour has to be 1, rightmost minute digit is in the range [3, 9] → **7 ways**

Leftmost minute digit = 3, Hour can be 1, 2 or 12, so three choices.
Right most minute digit is in range [4, 9] → 3 x 6 = **18 ways**

Leftmost minute digit = 4, Hour can be 1, 2, 3 or 12, so four choices.
Right most minute digit is in range [5, 9] → 4 x 5 = **20 ways**

Leftmost minute digit = 5, Hour can be 1, 2, 3, 4 or 12, so five choices.
Right most minute digit is in range [6, 9] → 5 x 4 = **20 ways**

Total = 7 + 18 + 20 + 20 = <u>65 ways</u>

Grading: Lots of ways to solve this.

If their method isn't clear, then we can simply award a grade by how close they get:

10 pts – 65,
9 pts – 64, 66,
7 pts – [60, 63] or [67. 70],
5 pts – [55, 59], [71, 75]
3 pts – [45, 54], [76, 85]
1 pt – [35, 44], [86, 95]

If there's a method that's easy to follow, just take off 1 point per error, capping at 10.

Note: We can probably BF the hour, or the minute as well. I am looking forward to any approach that doesn't try to write them all.

7) (10 pts) Consider a variant of the Monty Hall problem with 10 doors, of which 3 have cars behind them and 7 have goats. Once the contestant picks a door, two other doors are revealed to have goats behind them.

(a) (2 pts) What is your probability of winning a car if you stay with the door you originally chose?

Recall from the original problem, that given that you stay, your probability of winning is simply the chance that you initially picked "a correct door." In this case, that probability is 3 out of 10 since 3 of the 10 doors have cars behind them.

$\mathbf{\frac{3}{10}}$

Grading: 2 pts all or nothing for the answer, no work needed.

(b) (8 pts) What is your probability of winning a car if you randomly switch to one of the other unrevealed doors?

Now we must work out the probability given that our strategy is switching!

$\frac{3}{10}$ of the time, we picked a correct door. Since the host reveals 2 goats, the 7 remaining doors have behind them 2 cars and 5 goats (since your door has a car behind it). If you switch to one of these random doors, your chance of switching to a different car is $\frac{2}{7}$. Thus, the probability of picking a car followed by switching to a different car is $\frac{3}{10} \times \frac{2}{7} = \frac{6}{70}$. (We'll leave it like this for now…)

$\frac{7}{10}$ of the time, we picked a door with a goat. Since the host reveals 2 more goats, the 7 remaining doors have behind them 3 cars and 4 goats (since your door has a goat behind it). If you switch to one of these random doors, your chance of switching to a car is $\frac{3}{7}$. Thus, the probability of picking a goat followed by switching to a car is $\frac{7}{10} \times \frac{3}{7} = \frac{21}{70}$. (We'll leave it like this for now…)

Thus, the total probability of winning, given that our strategy is to switch is $\frac{6}{70} + \frac{21}{70} = \mathbf{\frac{27}{70}}$

Grading: **3 pts for first branch (decide partial as needed),**
4 pts for second branch (decide partial as needed),
1 pt for adding and getting the final answer, should be as a fraction.

8) (10 pts) Let X be a continuous random variable defined by the probability density function below:

$$p(x) = cx^2, 0 \le x \le 3$$
$$p(x) = 0, otherwise$$

(a) (3 pts) Find the value of c. Please express your answer as a fraction in lowest terms.

The total area under the curve must be 1. $\int_0^3 cx^2 dx = 1 \rightarrow \frac{c}{3}x^3|_0^3 = 1 \rightarrow 9c = 1 \rightarrow \boldsymbol{c} = \frac{\mathbf{1}}{\mathbf{9}}$

$\frac{\mathbf{1}}{\mathbf{9}}$**, Grading: 1 pt writing down integral equal to 1, 1 pt do integral, 1 pt plug in and get c**

(b) (3 pts) Determine the probability that 1 ≤ X ≤ 2. Please express your answer as a fraction in lowest terms.

$$p(1 \le x \le 2) = \int_1^2 \frac{1}{9}x^2 dx = \frac{1}{27}x^3|_1^2 = \frac{1}{27}(8-1) = \frac{\mathbf{7}}{\mathbf{27}}$$

$\frac{\mathbf{7}}{\mathbf{27}}$**, Grading: 1 pt writing down correct integral, 1 pt doing integral, 1 pt plugging in bounds, getting answer in lowest terms.**

(c) (4 pts) Determine the value of E(X). Please express your answer as a fraction in lowest terms.

$$E(X) = \int_0^3 (x)\frac{1}{9}x^2 dx = \int_0^3 \frac{1}{9}x^3 dx = \frac{1}{36}x^4|_0^3 = \frac{1}{36}(81-0) = \frac{81}{36} = \frac{\mathbf{9}}{\mathbf{4}}$$

Grading:
1 pt for setting up correct integral (with x multiplied in)
1 pt for doing the integral
1 pt for plugging in to get 81/36
1 pt to reduce to 9/4

9) (10 pts) Ava lives D miles away from work. When she drives to work at an average speed of 45 miles per hour, she arrives to work exactly on time. If she were to leave at the exact same time, but drive an average of 40 miles per hour, she would arrive at work two and a half minutes late.

(a) (6 pts) How many **miles** away from work does she live?

The time she should take to drive to work is $\frac{D}{45}$ hours. For the scenario given, she takes $\frac{D}{40}$ hours instead. The question tells us that this is exactly equal to

$\frac{D}{45}$ hours + 2.5 minutes = $\frac{D}{45} + \frac{2.5}{60} = \frac{D}{45} + \frac{5}{120} = \frac{D}{45} + \frac{1}{24}$, since Ava's 2.5 minutes late when she drives this speed. Set these two expressions equal to one another:

$$\frac{D}{40} = \frac{D}{45} + \frac{1}{24}$$

$\frac{D}{40} - \frac{D}{45} = \frac{1}{24}$, multiply through by 360 (LCM) to get

$$9D - 8D = 15$$
$$D = \mathbf{15\ miles}$$

Grading: 3 pts to arrive at equation in terms of D based on late factor
3 pts to solve the equation correctly
Give partial as needed

(b) (4 pts) If she were to leave at the exact same time but drive at an average speed of 50 miles per hour, how many minutes early would she arrive at work?

Reg time = $\frac{15\ miles}{45\ mph} \times \frac{60\ minutes}{1\ hour} = 15 \times \frac{4}{3} = 20\ minutes$

Fast time = $\frac{15\ miles}{50\ mph} \times \frac{60\ minutes}{1\ hour} = 15 \times \frac{6}{5} = 18\ minutes$

She would arrive 20 – 18 = **2 minutes early.**

Grading: 1 pt for regular time, 2 pts for early time, 1 pt for answeri

10) (10 pts) Elijah invests \$1000 dollars each year in an account that accrues 6% interest. He puts his investment in once a year at the same time. Thus, if he's made n investments (over a total duration of n – 1 years), we can think of the current amount of money he has invested as investing \$1000 for n – 1 years, investing another \$1000 for n – 2 years, investing another \$1000 for n – 3 years, … and investing the last \$1000 for 0 years.

(a) (1 pt) Write down the value of \$1000 invested for precisely k years (at 6% annual growth) in terms of k.

$\underline{\$1000(1.06)^k}$ (Grading: 1 pt all or nothing)

(b) (2 pts) Using the expression from (a), noting that the scenario described can be modeled as n separate investments, write down a summation equal to the total current value of Elijah's investment. (Hint: Your sum should go from k = 0 to k = n – 1.)

$$\sum_{k=0}^{n-1} \$1000(1.06^k)$$

Grading: 1 pt inside sum, 1 pt bounds

(c) (3 pts) Using the geometric sum formula, determine the value of the sum above in terms of n.

$$\frac{\$1000(1.06^n - 1)}{(1.06 - 1)} = (\$\mathbf{1000}) \times \frac{(\mathbf{1.06^n - 1})}{\mathbf{.06}}$$

Grading: 1 pt \$1000 factored out, 1 pt numerator final answer, 1 pt denominator final answer

(d) (4 pts) Using the result from part (c), determine the minimum number of consecutive annual $1000 investments Elijah must make so that the total value of his investment exceeds $100,000. In particular, for full credit, you must solve for this minimum value n and get an answer of the form $\log_a b$, where a is a real number greater than 1 and b is a positive integer. (Since n must be integral, the answer to the question will simply be the smallest integer greater than this expression.) Put a box around your answer. (You'll need a calculator for this part.)

$$(\$1000) \times \frac{(1.06^n - 1)}{.06} > \$100{,}000$$

$$\frac{(1.06^n - 1)}{.06} > 100$$

$$1.06^n - 1 > 6$$

$$1.06^n > 7$$

$$n > log_{1.06} 7 = \frac{\ln 7}{\ln 1.06} \sim 33.395$$

The answer to the question is n = 34, right after he makes the 34th consecutive annual investment (33 years after his very first investment), the total value of his investment will be roughly $104,183.75.

Grading: **1 pt setting up equation**
1 pt to get to $1.06^n > 7$
1 pt to write inequality for n in terms of a log
1 pt to get the correct integer final answer. No need to state value of Investment at that time.

11) (2 pts) Moana 2 recently opened in theaters across the US.
How many Moana movies have been made so far? **TWO (Give to all)**