COT 3100 Quiz #2 (Version A): Counting (7/8/2026) Solutions

1) (5 pts) **There are 10 girls and 7 boys in a class.** Answer the following counting questions about the class. Please give an integer answer for questions a, b and c. You may leave your answer for questions d and e in powers, factorials and combinations.

a) (1 pt) One student must be chosen as class president. In how many ways can the president be chosen?

17 **(1 pt)**

b) (1 pt) One boy and one girl must be chosen to represent the class at the school wide mixed-doubles ping-pong tournament. How many possible different teams could represent the class?

70 **(1 pt)**

c) (1 pt) The teacher creates teams of 2 with one girl and one boy, but these leaves some girls left out, who are all placed on their own team. How many girls are on this team?

3 **(1 pt)**

d) (1 pt) The teacher must line up the class. In how many different orders can she line up the students in the class?

17! **(1 pt)**

e) (1 pt) A teacher asks each student whether or not they like pizza. Each student answers either "YES" or "NO". The chalkboard shows that exactly 10 students said yes. How many possible subsets of students could have said "YES", based on this information?

$\binom{17}{10}$, other acceptable answers $\binom{17}{7}$, $\frac{17!}{7!10!}$, C(17,10), C(17,7), ${}_{17}C_{10}$ or ${}_{17}C_{7}$ or equivalent **(1 pt)**

2) (5 pts) At the time of this quiz, exactly 8 teams remain vying for the World Cup. There will be four quarterfinal matches, two semifinal matches and one final match. (The 4 winning teams in the quarterfinals play in the semifinals and then the two winning teams in the semifinals play in the finals.) There are a possible 128 possible outcomes amongst all of the matches. How many of these possible outcomes involve France facing Norway in the final match? **Note: France and Norway are on opposite sides of the draw, so they CAN NOT face each other until the final match, if at all.** Please express your answer as a single integer.

France must win its first two matches (so only 1 outcome here). Norway must win its first two matches (so only 1 outcome here). There are three matches that are allowed to go either way, two quarterfinal matches not involving either team and the final. Each of these matches have 2 possible outcomes. It follows that there are 2^3 = **8** possible outcomes which involve a final match between France and Norway. **(Grading: 5 pts for correct answer, 3 pts for an answer of the form 2^x,**

For the rest of the problems, please leave your answers in powers, factorials and combinations.

3) (10 pts)
(a) (2 pts) How many permutations of the letters in LIONKING are there?

$\frac{8!}{2!2!}$, **1 pt numerator, 1 pt denominator, 0 pts if answer isn't in the right format**

(b) (4 pts) How many permutations of the letters in LIONKING start AND end with a vowel?

There are 3 vowels, two Is and an O. First count all permutations that start and end with I: there are 6 other letters, with 2 Ns repeated in this case, which amounts to $\frac{6!}{2!}$ permutations that start and end with I. Next count permutations that start with I and end with O. There are 6 letters left to permute, again with just 2 Ns and the rest of the letters distinct, so there are $\frac{6!}{2!}$ of these permutations as well. Finally, there will also be $\frac{6!}{2!}$ permutations that start with O and end with I, since the middle six letters are the same as the previous case. Adding these cases we get $\frac{\mathbf{3(6!)}}{\mathbf{2!}}$. This is small enough to work out, it's 1080 different permutations of LIONKING that start and end with a vowel.

Grading: 1 pt for each case, 1 pt for adding properly. No need to simplify to 1080.

(c) (4 pts) How many permutations of the letters in LIONKING do NOT have two consecutive vowels?

Use the 5 consonants as separators: __ L __ N __ K __ N __ G __. This leaves six slots to place the vowels choosing at most 1 vowel per slot. We can do this in $\binom{6}{3}$ ways. Once we choose the slots, we can permute the 2 Is and O in 3 ways ($\frac{3!}{2!}$). Finally, we can permute the 5 consonants in $\frac{5!}{2!}$ ways. Since each of these three choices are make independently of the others and we can match any option for one choice with any option for another choice, we multiply each of these to get our final answer: $\binom{6}{3} \times 3 \times \frac{5!}{2!} = 20 \times 3 \times 60 = \mathbf{3600}$**.**

Grading: 1 pt C(6, 3), 1 pt 3, 1 pt 5!/2!, 1 pt mult all (no need to get to 3600)

4) (5 pts) Jenna has chosen a passcode. Her passcode is a sequence of integers, where each integer is in between 1 and 100, with no repeats. It just so happens that Jenna's passcode contains 4 integers and is in increasing order. Stella is trying to break Jenna's passcode by trying each possibility based on this information. In the worst case, how many sequences must Stella try to find Jenna's passcode?

$\binom{100}{4}$, since each combination of 4 integers in range maps to precisely one valid ascending sequence. **Grading: 5 pts correct, 3 pts wrong combo, 0 otherwise. (Make sure to accept all alternate forms.)**