COT 3100 Recitation Problems: Induction

1) Use induction on n to prove the following summation for all positive integers n:

$$\sum_{i=1}^{n} i^3 = \frac{n^2(n+1)^2}{4}$$

2) The n^{th} Harmonic number, denoted H_n is defined as follows: $H_n = \sum_{i=1}^{n} \frac{1}{i}$

Prove that the following equation is true for all positive integers n, using induction on n:

$$\sum_{i=1}^{n} \frac{i}{i+1} = (n+1) - H_{n+1}$$

3) Use induction on n to prove the following inequality for all positive integers n:

$$\sum_{i=0}^{n} 3^i < \frac{3^{n+1}}{2}$$

4) The Fibonacci numbers are defined as follows: $F_0 = 0$, $F_1 = 1$, $F_n = F_{n-1} + F_{n-2}$, for all integers $n>1$. Prove the following formula for all positive integers n:

$$\sum_{i=1}^{n} \frac{F_{i-1}}{2^i} = 1 - \frac{F_{n+2}}{2^n}$$

5) Using induction on n, prove for all non-negative integers n that $9 \mid (2^{2n} + 6n - 1)$.

6) Define the sequence t_n as follows: $t_0 = 7$, $t_1 = 10$, $t_n = 3t_{n-1} - 2t_{n-2}$, for all integers $n \geq 2$. Prove, using strong induction on n with 2 base cases, that for all non-negative integers n,

$$t_n = 4 + 3(2^n).$$

7) Prove, for all positive integers n, using induction on n that

$$\begin{bmatrix} 2 & -1 \\ -3 & 4 \end{bmatrix}^n = \frac{1}{4}\begin{bmatrix} 5^n + 3 & 1 - 5^n \\ 3(1 - 5^n) & 3(5^n) + 1 \end{bmatrix}.$$