COT 3100 Recitation Problems: Induction Prep (Sums, Matrices, Rec. Rel)

1) An arithmetic sequence $a_1, a_2, a_3, \ldots$ is such that $a_{10} = 40$ and $a_{25} = 10$. Determine both a_1 and the sum of the first 50 terms of the sequence.

2) An infinite geometric sequence $a_1, a_2, a_3, \ldots$ has a sum of 10. If the odd indexed terms (a_1, a_3, etc.) have a sum of 7, what is the common ratio of the sequence? What is the first term of the sequence?

3) Determine the following summation in terms of n: $\sum_{i=n+1}^{2n}(3i - 2)$.

4) Determine the following summation in terms of n: $\sum_{i=1}^{2n}(3i^2 - 4i)$.

5) Determine the following infinite summation: $\sum_{i=1}^{\infty}(2i - 1)(\frac{1}{2})^i$.

6) Determine the following matrix sum: $\begin{bmatrix} 6 & 3 & -2 \\ 5 & -9 & 7 \end{bmatrix} + \begin{bmatrix} -2 & 12 & 3 \\ 6 & 4 & 8 \end{bmatrix}$.

7) Determine the following matrix product, $\begin{bmatrix} 2^n & 1 - 2^n \\ 3 & 2^{n-1} \end{bmatrix}\begin{bmatrix} 3 & 2 \\ 1 & -1 \end{bmatrix}$, in terms of n.

8) Let the Fibonacci Sequence be defined as follows: $F_0 = 0, F_1 = 1, F_n = F_{n-1} + F_{n-2}$, for all integers $n \geq 2$. Determine F_{n+2} in terms of F_{n-1} and F_{n-2}.

9) Once again, let F_n denote the n^{th} Fibonacci number (defined in the previous question). Determine and simplify the following matrix product, in terms of n:

$$\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}\begin{bmatrix} F_n & F_{n-1} \\ F_{n-1} & F_{n-2} \end{bmatrix}$$

10) Let a, b, x and y be arbitrary positive constants. Let $f(n) = ax^n + by^n$ for all non-negative integers, n.

For an arbitrary positive integer, n, prove that

$f(n+1) = (x+y)f(n) - xyf(n - 1)$.