COT 3100 Recitation Problems: Number Theory

1) Find the quotient and remainder for the following division problems:

i) $a = 173$, $b = 9$
ii) $a = 54321$, $b = 103$
iii) $a = 177$, $b = 496$
iv) $a = 429$, $b = 37$
v) $a = 2018$, $b = 162$

2) Without the aid of a calculator, find the value of the remainders for each of the following divisions:

i) $a = 46^4$, $b = 17$
ii) $a = 10103^4$, $b = 10$
iii) $a = 67^{10}$, $b = 20$
iv) $a = 1079^{14}$, $b = 98$

3) Convert each of the following values to base 10:

i) $3F2A_{16}$
ii) 417_8
iii) 244015_5
iv) 10001110111_2

4) Prove the following: Let the remainder when a is divided by b be r, where $a, b > 0$ and $0 \le r < b$. Show that $a^2 \equiv r^2 \pmod{b}$, formally. That is, prove that $b \mid (a^2 - r^2)$.

5) In base r, 1000 - 340 equals 440. What is r?

6) Find all integer solutions to the equation $1141x + 406y = 28$.

7) (a)Find all integer solutions to the equation $105x + 83y = 1$.

(b) Find all integer solutions to the equation $105x + 83y = 8$.

(c) Find $83^{-1} \bmod 105$. (Note: Answer must be in between 0 and 104, inclusive.)

8) Let x and y be integers such that $15 \mid (3x + 4y)$. Prove that $15 \mid (12x + y)$.

9) Let $a = 2^4 3^2 5^6 7^8$, $b = 2^2 3^9 5^4 11^5$, and $c = 2^5 3^7 5^3 11^2$. Determine, in prime factorized form, both $\gcd(a, b, c)$ and $\mathrm{lcm}(a, b, c)$.

10) For the numbers a, b and c listed in problem 9, determine the number of divisors each of those numbers has.