Recitation Problems: Logic

Note: The intention isn't necessarily for groups to finish these problems during the 50 minutes of recitation, but rather, the opposite – to provide enough practice that all groups will fully utilize all 50 minutes of recitation and if students want, can continue to work on the problems after recitation is over.

1) Create a truth table for the following proposition. Include columns for intermediate parts.

$$(p \wedge \bar{q}) \rightarrow (r \vee \bar{p})$$

2) Below are the truth tables for NAND, NOR, and XOR ($\oplus$). Write definitions for these operators in terms of AND, OR, and NOT. Try to explain in words what each one means.

A	B	A NAND B	A NOR B	A $\oplus$ B
F	F	T	T	F
F	T	T	F	T
T	F	T	F	T
T	T	F	F	F

3) The operator NAND, described above, can be called "computationally complete," or a "sole sufficient operator." That means that by itself, it can replace any of the other logical operators. How would you rewrite the statement "¬P" using only NAND?

4) Prove the following using the laws of logic and the implication identity:

$$(\ (p \vee \neg p)\ \wedge\ (\ q \vee\ \neg(\neg q \vee \neg r)\ )\ )\ \vee\ (\ (p \vee\ \neg p)\ \wedge\ \neg q) \leftrightarrow T$$

5) Prove the following using the laws of logic and the implication identity:

$$(\neg q \rightarrow \neg p) \rightarrow ((p \wedge q) \vee r)) \leftrightarrow r \vee p$$

6) Using the following premises:

$(p \wedge t) \rightarrow (r \vee s)$
$q \rightarrow (u \wedge t)$
$u \rightarrow p$
$\bar{s}$

Derive the conclusion $q \rightarrow r$.

7) In class, Modus Ponens was proved using just the laws of logic. Prove Modus Tollens in the same manner.

8) Find your own open numerical statement (with the universe of positive integers), $P(x, y)$ and $Q(x)$ such that exactly one of $\forall x(\exists y | P(x, y))$ and $\exists x(\forall y | P(x, y))$ is true.

9) In the course text (and later in class), a proof was shown indicating that $\sqrt{2}$ is an irrational number. Explain why "repeating" the proof for the $\sqrt{3}$ succeeds but trying it for $\sqrt{9}$ fails.

10) Prove or disprove the following statements over the universe of real numbers:

a) $\exists x \forall y(xy = y)$ b) $\forall x \exists y(3y^2 - 2y + 6 = x)$ c) $\forall x \forall y \exists z((z > x) \wedge (z < y))$