COT 3100 7/7/2026

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Today: Counting Lecture, H6 is posted → Counting + Prob

Thursday: DO **NOT** COME TO LECTURE!

WATCH 2 ZOOM VIDEOS

(a) Sample Counting Problems

(b) Intro to Probability

## #9 exam 2

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Count all strings length $k+1$ that don't have "bb".

$\underbrace{\qquad\qquad\qquad}_{}$ $k$ → any string w/o bb of length $k$ can be placed here!

____________ a $\qquad F_{k+2}$

____________ a b

$k-1$ chars — a must be 2nd to last $\qquad +$

anything that doesn't have bb $F_{k+1}$ $\longrightarrow F_{k+3}$

# Combinations w/Repetition

M = Miller, B = Bass, N = Natty, S = Sam, K = Killians

$$M + B + N + S + K = 8$$

$$M, B, N, S, K \geq 0, \quad M, B, N, S, K \in \mathbb{Z}$$

$$| * * * | * * | * | * *$$

$$\begin{array}{ccccc} 0 & 3 & 2 & 1 & 2 \\ M & B & N & S & K \end{array}$$

For each arrangement of 8 stars, 4 bars maps to a unique beer order.

Similarly any beer order maps to 1 arrangement of stars + bars

$$3M, 0B, 4N, 1S, 0K$$

$$* * * | | * * * * | * |$$

Answer is # ways to arrange 8 $*$s, 4 $|$s.

$$\frac{12!}{8!4!}, \quad \_\ \_\ \_\ |\ |\ \_\ \_\ \_\ \_\ \_\ |\ \_\ |$$

perms

$\binom{12}{4} \to$ choose 4 slots out of 12 bars

$\binom{12}{8} \to$ choose 8 slots out of 12 stars

$$\frac{(n+r-1)!}{n!(r-1)!}$$

Buying $n$ objects

Exist $r$ different kinds

$$\binom{n+r-1}{r-1} = \binom{n+r-1}{n}$$

## Minimum Requirement

M, B, N, S, K, 8 6-packs

must buy at least 3 6-packs of sam.

$M+B+N+S+K=8$ but $S \geq 3$

$M+B+N+S'+3+K=8$ let $S' = S-3$

$S = S'+3$

$M+B+N+S'+K=5$ $S' \geq 0$

$n=5$ (buy 5)

$r=5$ (5 kinds)

$$\binom{5+5-1}{5-1} = \binom{9}{4}$$

## Maximum Requirement

Go to Store only have 2 6-packs of Sam ☹

$M+B+N+S+K=8$ but $S \leq 2$

ALL SOLS − ($S \geq 3$)

$$\binom{12}{4} - \binom{9}{4}$$

## 2 Max Requirements

$S \le 2$ and $B \le 3$

$$M+B+N+S+K=8$$

**$S \ge 3$**

$$\binom{9}{4}$$

**$B \ge 4$**

buy 4 $B'$ is extra

$M+B'+N+S+K=4$

$n=4,\ r=5$ $\binom{4+5-1}{5-1}=\binom{8}{4}$

**Double counted $S \ge 3$ and $B \ge 4$**

$M+B'+N+S'+K=1$ buy 3 sam, 4 bass

$$n=1,\ r=5,\ \binom{1+5-1}{5-1}=\binom{5}{4}$$

# combos with $S \ge 3$ or $B \ge 4$ is

$$\binom{9}{4}+\binom{8}{4}-\binom{5}{4}$$

**Total answer**

$$\binom{12}{4}-\left[\binom{9}{4}+\binom{8}{4}-\binom{5}{4}\right]$$

$$\binom{12}{4}-\binom{9}{4}-\binom{8}{4}+\binom{5}{4}$$

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how to work out a combination by hand

$$\binom{12}{4}=\frac{12!}{4!\,8!}=\frac{\cancel{12}\times 11\times \overset{5}{\cancel{10}}\times 9}{1\times\cancel{2}\times\cancel{3}\times\cancel{4}}=45\times 11=495$$

## Inequality

$$M + B + N + S + K \leq 8$$

$$(3, 1, 0, 0, 0) \longleftrightarrow (3, 1, 0, 0, 0, 4)$$

$$M + B + N + S + K + Z = 8$$

slack variable all beer you chose not to buy!

$n = 8, \quad r = 6$

$$\binom{\cancel{\phantom{x}}\, 8+6-1}{6-1} = \binom{13}{5}$$

## Martian Life (from typed notes)

(a) # valid commands

___ ___ ___ 3 words

$3^4$ possible words — each word is 4 letters

$3^4 \times 3^4 \times 3^4 = 3^{12}$ — letters $= \{a, b, c\}$

(b) distinct words in command

order doesn't matter

$3^4 = 81$ possible words $\binom{81}{3}$

any choice of 3 is a valid command

# Counting Numbers

(a) 6 digit integers, distinct digits, don't start w/ 0

$$\underset{\substack{\downarrow \\ \text{any but} \\ 0}}{\underline{9}} \times \underset{\substack{\downarrow \\ \text{any} \\ \text{but} \\ \text{1st} \\ \text{choice}}}{\underline{9}} \times \underline{8} \times \underline{7} \times \underline{6} \times \underline{5} = \frac{9 \times 9!}{4!}$$

(b) How many contain 3 but <u>NOT</u> 6

$$3\ \underline{\ }\ \underline{\ }\ \underline{\ }\ \underline{\ }\ \underline{\ }$$
$$8 \times 7 \times 6 \times 5 \times 4$$

$$\underline{\ }\ \underline{\ }\ \underline{\ }\ \underline{\ }\ 3\ \underline{\ } \quad \} \text{ 3 in any of 5 last slots}$$
$$7 \times 7 \times 6 \times 5 \times 4$$

7
can't do 0, 3, 6

$$\text{final ans} = 8 \times \boxed{7 \times 6 \times 5 \times 4} + \underset{\substack{\uparrow \\ \text{placement of 3}}}{\underline{5}} \times 7 \times \boxed{7 \times 6 \times 5 \times 4}$$

$$= 7 \times 6 \times 5 \times 4\,(8 + 35)$$

$$= 42 \times 20 \times 43$$

(c) How many contain 3 or 6 or both?

WHOLE − (Don't have either)

$$\frac{9 \times 9!}{4!} - \frac{7 \times 7!}{2!}$$

$$\underline{\ }\ \underline{\ }\ \underline{\ }\ \underline{\ }\ \underline{\ }\ \underline{\ }$$
$$7 \times 7 \times 6 \times 5 \times 4 \times 3$$

## Permutations

(a) COMBINATORICS    13 letters    $\frac{13!}{2!2!2!}$

2 Cs, 2 Os, 2 Is

(b) all vowels in a block

$\boxed{OIAOI}$, ~~2Cs~~ 2 Cs, M, B, N, T, R, S

$$\frac{9!}{2!} \times \frac{5!}{2!2!}$$

orders w/ fixed superletter    # of options for superletter

(c) BACICIENOONRST

B A C I C I N O O N R S T    13 slots

Choose 5 for vowels    $\boxed{\binom{13}{5}}$ ways

## More Perm

# perm ABCDEF

A before C, E before C

A E

_ E A _ _ C    2 of these work out of 6 total

3!

$\frac{1}{3} \times 6! = 240$    Choose 3 locations for A, C, E out of 6

$20 \times 2 \times 6 = \underline{240}$    $\binom{6}{3} \times 2 \times 6 \to$ # valid orders of B, D, F

# Even More Perm

a) Perm FOUNDATION $\rightarrow \dfrac{10!}{2!2!}$

2 Os, 2 Ns

b) 5 letter passwords

2 Os 2 Ns, 6 other letters

No repeat $\longrightarrow$ 8 letters, $8\times7\times6\times5\times4$, $P(8,5)$

$$\frac{8!}{3!}$$

2 Ns, rest distincts } choice of letters is $

_ N _ _ N

$\binom{5}{2} \times 7 \times 6 \times 5$

where NS go — 1st slot, 2nd slot, 3rd slot

$\cancel{450}$
$\cancel{42}$
$\cancel{900}$
$\cancel{1800}$
$\cancel{18900}$

**2 Os**, rest distinct same as above

2 Ns, 2 Os, 6 choises last letter

$$6 \times \frac{5!}{2!2!}$$

$$\frac{8!}{3!} + \left(\binom{\cancel{10}\,5}{2} \times 7\times6\times5\right) \times 2 + 6 \times \frac{5!}{2!2!}$$

$$\frac{40320}{6} + \cancel{45}\,10 \times 42 \times \cancel{5\times2}\,10 + \frac{6\times120}{4}$$

$6720 + \cancel{18900}$ 4200 $+ 180$ ✓ $= 11{,}100$

6720
4200
180
11100

Ice Cream

10 flavors
7 toppings

(a) $\le 2$ scoops single flavor
exactly 4 scoops
upto 5 toppings (at least 1 topping)

flavor

| | |
|---|---|
| 1,1,1,1 | $\binom{10}{4}$ |
| 1,1,2 | $10 \times \binom{9}{2}$ |
| 2,2 | $\binom{10}{2}$ |

$$\left(\binom{10}{4} + 10 \times \binom{9}{2} + \binom{10}{2}\right)\left(2^7 - \underset{\substack{\uparrow \\ \text{no toppings}}}{1} - \underset{\substack{\uparrow \\ \text{7 topping}}}{1} - \underset{\substack{\uparrow \\ \text{6 toppings}}}{\binom{7}{6}}\right)$$

(b) 10 Flavors
7 toppings

7 orders [1 F, 1 T]
all distinct!

$t_1 \to 10$
$t_2 \to 9$
$t_3 \to 8$

$t_7 \to 4$

$$\frac{10!}{3!}$$