Summer 2026 COT 3100 Homework #4

1) Use induction on n to prove for all positive integers n that

$$\sum_{i=1}^{n} i^3 = \frac{n^2(n+1)^2}{4}$$

2) Let the sequence t_n be defined as follows:

$$t_0 = 1, t_1 = 4, t_n = 5t_{n-1} - 6t_{n-2}$$

Use trial and error to guess a closed form solution for t_n, in terms of n. (For example, something like $t_n = 2n^2 + n + 1$.) Then, use strong induction on n with two base cases to prove that your formula is correct for all non-negative integers n.

3) Prove, using induction on n, that $16 \mid (7^n + 9^n + (-1)^{n+1} + 15)$ for all non-negative integers n.

4) Prove, via strong induction on n, for all integers $n \geq 24$, that you can buy exactly n chicken nuggets given that you can purchase any number of 5-nugget packs and any number of 7-nugget packs. (Note: you must determine the number of base cases that are necessary.)

5) Prove for all positive integers n, via induction on n, that

$$\sum_{i=1}^{n^3} \sqrt[3]{i} \leq \frac{n^2(n+1)(3n-1)}{4}$$