

Summer 2026 COT 3100 Homework #1

Note: Homework Assignments will be graded on a 5 point scale, where each grade means the following, as interpreted by the grader:

- 5 – Nearly all correct, no significant logical errors**
- 4 – At least 80% of questions answers and at most minor logical errors**
- 3 – At least half done and most of what is done is correct**
- 2 – Not half done or strictly less than half correct, significant holes in understanding**
- 1 – Submitted, but didn't seem to deserve a 2. (No submission is 0.)**

By reducing the amount of time graders spend trying to decide the grade, they can spend more time providing qualitative feedback. That is the aim of this simplified grading system.

Note: please type your homework in either Word using equation editor or LaTeX. The final submission extension must be .pdf, so save your work in a .pdf file to submit. Maximum grade for any handwritten scanned homework is 3 out of 5.

1) Create a truth table for the following Boolean expression. Your table should have eight rows and at least 7 columns (the first three columns should be labeled p , q and r , and the last column should be labeled with the full expression).

$$(p \vee \bar{q}) \wedge \overline{(r \vee (q \wedge p))}$$

2) Below is a truth table with four columns: p , q , r , and a mystery column. Though any method you would like, determine a valid logical expression equivalent to the last column of the truth table. Try to minimize the length of your expression. (There's a trivial solution that produces a fairly long expression.)

p	q	r	Unknown Expression
F	F	F	F
F	F	T	T
F	T	F	T
F	T	T	F
T	F	F	T
T	F	T	F
T	T	F	T
T	T	T	T

3) Find the values of each of the following bitwise operations. Please show your bit-by-bit work for each calculation.

- (a) $87 \& 49$
- (b) $83 | 38$
- (c) $39 \wedge 74$
- (d) $202 \gg 4$
- (e) $18 \ll 3$

4) Use the truth table method to prove the following expression is a tautology:

$$(p \rightarrow q) \vee (p \rightarrow r) \vee ((r \wedge \bar{q}) \vee \bar{q})$$

5) Use the laws of logic to prove the following expression is a tautology. Show the application of EVERY STEP, including the Commutative and Associative Laws.

$$(p \rightarrow q) \vee (p \rightarrow r) \vee ((r \wedge \bar{q}) \vee \bar{q})$$

6) Use the rules of inference to prove the following argument. Make sure to show EVERY STEP, even steps like Disjunctive Amplification.

$$\begin{array}{l} p \rightarrow r \\ q \rightarrow r \\ \bar{r} \\ \bar{p} \rightarrow s \\ q \wedge \bar{t} \\ \hline \therefore s \wedge t \end{array}$$

7) Use the rules of inference to prove the following argument. Make sure to show EVERY STEP, even steps like Disjunctive Amplification.

$$\begin{array}{l} \bar{p} \vee q \\ \bar{q} \\ r \rightarrow p \\ r \vee s \vee t \\ t \rightarrow r \\ \hline \therefore s \end{array}$$

8) Prove or disprove the following claim over the universe of all **positive** real numbers for x and y.

$$\forall x[\exists y[x + y = xy]]$$

If the claim is false, find a value of x for which it is false. If it is true, show which value(s) of y exist to make the claim true for all x.

9) Prove that for all real numbers x, y and z, that

$$x^2 + y^2 + z^2 \geq xy + xz + yz$$

Hint: Since all real numbers squared are non-negative, it's true that

$$(x - y)^2 + (y - z)^2 + (z - x)^2 \geq 0$$

10) Let the Boolean variable p be set to true if the integer a is odd, and false if a is even. Similarly, let the Boolean variable q be set in a similar manner according to integer b, r be set in similar manner according to integer c and s be set in a similar manner according to integer d.

With proof, for how many truth settings of p, q, r and s (out of 16) is the algebraic expression

$$ab + cd$$

equal to an even number?