Summer 2026 COT 3100 Homework #1 Solutions

1) Create a truth table for the following Boolean expression. Your table should have eight rows and at least 7 columns (the first three columns should be labeled p, q and r, and the last column should be labeled with the full expression).

$$(p \vee \bar{q}) \wedge (\overline{r \vee (q \wedge p)})$$

Solution

p	q	r	$p \vee \bar{q}$	$q \wedge p$	$\overline{r \vee (q \wedge p)}$	$(p \vee \bar{q}) \wedge (\overline{r \vee (q \wedge p)})$
F	F	F	T	F	T	T
F	F	T	T	F	F	F
F	T	F	F	F	T	F
F	T	T	F	F	F	F
T	F	F	T	F	T	T
T	F	T	T	F	F	F
T	T	F	T	T	F	F
T	T	T	T	T	F	F

Note: If we scan the right-most column, we can see that this full expression is equivalent to $\bar{q} \wedge \bar{r}$.

2) Below is a truth table with four columns: p, q, r, and a mystery column. Though any method you would like, determine a valid logical expression equivalent to the last column of the truth table. Try to minimize the length of your expression. (There's a trivial solution that produces a fairly long expression.)

p	q	r	Unknown Expression
F	F	F	F
F	F	T	T
F	T	F	T
F	T	T	F
T	F	F	T
T	F	T	F
T	T	F	T
T	T	T	T

Solution

Here is one possible expression, which uses the XOR operator: $[p \wedge (q \vee \bar{r})] \vee [\bar{p} \wedge (q \oplus r)]$

To create the first part of the expression, it was noted that 3 out of the four entries on the table with p set to True are set to true for the unknown expression, so we can "knock these three out" by "anding" p with the condition opposite to $\bar{q} \wedge r$, the settings on the one row on the bottom half of the truth table where the unknown expression is false.

To get the second part, it was noted that exactly the two rows where exactly one of q and r were true in the top half were the two rows where the unknown expression was set to true. This is precisely what XOR checks for, so the expression was completed.

There are many possible correct solutions. The goal is to make the final expressions shorter than the or of five clauses, each with 3 terms anded together.

3) Find the values of each of the following bitwise operations. Please show your bit-by-bit work for each calculation.

(a) 87 & 49
(b) 83 | 38
(c) 39 ^ 74
(d) 202 >> 4
(e) 18 << 3

<u>**Solution**</u>

For each of these questions, first convert each of the numbers to binary. Since this wasn't formally taught in class, you may either intuitively sum up powers of 2, or use the proper repeated division by 2 technique. (Had I taught it, the latter would be required.

(a)

<table>
<tr><td>2 | 87</td><td>2 | 49</td><td>1010111</td></tr>
<tr><td>2 | 43 R 1</td><td>2 | 24 R 1</td><td>&0110001</td></tr>
<tr><td>2 | 21 R 1</td><td>2 | 12 R 0</td><td>----------</td></tr>
<tr><td>2 | 10 R 1</td><td>2 | 6 R 0</td><td><u>0010001 (17)</u></td></tr>
<tr><td>2 | 5 R 0</td><td>2 | 3 R 0</td><td></td></tr>
<tr><td>2 | 2 R 1</td><td>2 | 1 R 1</td><td></td></tr>
<tr><td>2 | 1 R 0</td><td>110001 (read reverse)</td><td></td></tr>
<tr><td>1010111 (read reverse)</td><td></td><td></td></tr>
</table>

(b)

<table>
<tr><td>2 | 83</td><td>2 | 38</td><td>1010011</td></tr>
<tr><td>2 | 41 R 1</td><td>2 | 19 R 0</td><td>| 0100110</td></tr>
<tr><td>2 | 20 R 1</td><td>2 | 9 R 1</td><td>----------</td></tr>
<tr><td>2 | 10 R 0</td><td>2 | 4 R 1</td><td><u>1110111 (119)</u></td></tr>
<tr><td>2 | 5 R 0</td><td>2 | 2 R 0</td><td></td></tr>
<tr><td>2 | 2 R 1</td><td>2 | 1 R 0</td><td></td></tr>
<tr><td>2 | 1 R 0</td><td>100110 (read reverse)</td><td></td></tr>
<tr><td>1010011 (read reverse)</td><td></td><td></td></tr>
</table>

(c)

2 | 74
2 | 37 R 0
2 | 18 R 1
2 | 9 R 0
2 | 4 R 1
2 | 2 R 0
2 | 1 R 0
1001010 (read reverse)

2 | 39
2 | 19 R 1
2 | 9 R 1
2 | 4 R 1
2 | 2 R 0
2 | 1 R 0
100111 (read reverse)

```
  0100111
^ 1001010
  ----------
  1101101 (109)
```

(d)

2 | 202
2 | 101 R 0
2 | 50 R 1
2 | 25 R 0
2 | 12 R 1
2 | 6 R 0
2 | 3 R 0
2 | 1 R 1
11001010 (read reverse)

11001010 >> 4 = **<u>1100 (12)</u>**

(e)

2 | 18
2 | 9 R 0
2 | 4 R 1
2 | 2 R 0
2 | 1 R 0
10010 (read reverse)

10010 << 3 = **<u>10010000 (144)</u>**

4) Use the truth table method to prove the following expression is a tautology:

$$(p \to q) \vee (p \to r) \vee ((r \wedge \bar{q}) \vee \bar{q})$$

<u>Solution</u>

p	q	r	$p \to q$	$p \to r$	$(r \wedge \bar{q}) \vee \bar{q}$	$(p \to q) \vee (p \to r) \vee ((r \wedge \bar{q}) \vee \bar{q})$
F	F	F	T	T	T	T
F	F	T	T	T	T	T
F	T	F	T	T	F	T
F	T	T	T	T	F	T
T	F	F	F	F	T	T
T	F	T	F	T	T	T
T	T	F	T	F	F	T
T	T	T	T	T	F	T

5) Use the laws of logic to prove the following expression is a tautology. Show the application of EVERY STEP, including the Commutative and Associative Laws.

$$(p \to q) \lor (p \to r) \lor ((r \land \bar{q}) \lor \bar{q})$$

Solution

1. $(p \to q) \lor (p \to r) \lor ((r \land \bar{q}) \lor \bar{q})$	Given
2. $(\bar{p} \lor q) \lor (\bar{p} \lor r) \lor ((r \land \bar{q}) \lor \bar{q})$	Definition of Implication (applied 2 times)
3. $(\bar{p} \lor q) \lor (\bar{p} \lor r) \lor (\bar{q} \lor (r \land \bar{q}))$	Commutative Law
4. $(\bar{p} \lor q) \lor (\bar{p} \lor r) \lor (\bar{q} \lor (\bar{q} \land r))$	Commutative Law
5. $(\bar{p} \lor q) \lor (\bar{p} \lor r) \lor \bar{q}$	Absorption Law
6. $(\bar{p} \lor (q \lor (\bar{p} \lor r)) \lor \bar{q}$	Associative Law
7. $(\bar{p} \lor ((\bar{p} \lor r) \lor q) \lor \bar{q}$	Commutative Law
8. $((\bar{p} \lor (\bar{p} \lor r)) \lor q) \lor \bar{q}$	Associative Law
9. $(\bar{p} \lor (\bar{p} \lor r)) \lor (q \lor \bar{q})$	Associative Law
10. $(\bar{p} \lor (\bar{p} \lor r)) \lor T$	Inverse Law
11. T	Domination Law

I've been fairly explicit in applying Commutative and Associative. If you understand these laws, you can apply them multiple times in one step, but make sure to be careful. This only works if ALL of the operators in between the operands are the same (all ORs or all ANDs).

6) Use the rules of inference to prove the following argument. Make sure to show EVERY STEP, even steps like Disjunctive Amplification.

$p \to r$
$q \to r$
$\bar{r}$
$\bar{p} \to s$
$u \land \bar{t}$

$\therefore s \land \bar{t}$

Solution

1. $p \to r$	Given
2. $\bar{r}$	Given
3. $\bar{p}$	Modus Tollens with #1 and #2
4. $q \to r$	Given
5. $\bar{q}$	Modus Tollens with #4 and #2 (Not need but still valid)
6. $\bar{p} \to s$	Given
7. s	Modus Ponens with #6 and #3
8. $u \land \bar{t}$	Given
9. $\bar{t}$	Rule of Conjunctive Simplification with #8
10. $s \land \bar{t}$	Rule of Conjunction with #7 and #9

7) Use the rules of inference to prove the following argument. Make sure to show EVERY STEP, even steps like Disjunctive Amplification.

$\bar{p} \vee q$
$\bar{q}$
$r \rightarrow p$
$r \vee s \vee t$
$t \rightarrow r$

$\therefore s$

Solution

1. $\bar{p} \vee q$	Given
2. $q \vee \bar{p}$	Commutative Law with #1
3. $\bar{q}$	Given
4. $\bar{p}$	Disjunctive Syllogism with #2 and #3
5. $r \rightarrow p$	Given
6. $\bar{r}$	Modus Tollens with #5 and #4
7. $t \rightarrow r$	Given
8. $\bar{t}$	Modus Tollens with #7 and #6
9. $(r \vee s) \vee t$	Given
10. $s \vee t$	Disjunctive syllogism with #9 and #6
11. $t \vee s$	Commutative Law with #10
12. s	Disjunctive Syllogism with #11 and #8

8) Prove or disprove the following claim over the universe of all **positive** real numbers for x and y.

$$\forall x\big[\exists y[x + y = xy]\big]$$

If the claim is false, find a value of x for which it is false. If it is true, show which value(s) of y exist to make the claim true for all x.

Solution

The claim is not true. For x = 1, there is no value of y which makes the equation true. To see this, substitute x = 1 to get 1 + y = y. There's no value of y for which this equation is true, since the left-hand side must always be one greater than the right hand side, so the two sides could never be equal.

9) Prove that for all real numbers x, y and z, that

$$x^2 + y^2 + z^2 \geq xy + xz + yz$$

Hint: Since all real numbers squared are non-negative, it's true that

$$(x - y)^2 + (y - z)^2 + (z - x)^2 \geq 0$$

Solution

Start with the hint, which we know to be true because of the reason given above:

$$(x - y)^2 + (y - z)^2 + (z - x)^2 \geq 0$$

$$x^2 - 2xy + y^2 + y^2 - 2yz + z^2 + z^2 - 2xz + x^2 \geq 0$$

$$2x^2 + 2y^2 + 2z^2 - 2xy - 2xz - 2yz \geq 0$$

$$2x^2 + 2y^2 + 2z^2 \geq 2xy + 2xz + 2yz$$

$$x^2 + y^2 + z^2 \geq xy + xz + yz$$

This proves the assertion we were asked to prove. The creative step is starting with the known statement. Once a statement is known, then we're allowed to apply our usual algebra rules to the equation. This can NOT be done on an unknown statement.

10) Let the Boolean variable p be set to true if the integer a is odd, and false if a is even. Similarly, let the Boolean variable q be set in a similar manner according to integer b, r be set in similar manner according to integer c and s be set in a similar manner according to integer d.

With proof, for how many truth settings of p, q, r and s (out of 16) is the algebraic expression

ab + cd

equal to an even number?

Solution

The question is equivalent to asking how many settings of a, b, c and d make the expression above even when all four variables are limited to equal 0 or 1. To be even, both products must be the same parity, thus, either both ab and cd are even, OR both ab and cd are odd. The latter is easiest to count. In order for a product of integers to be odd, both integers must be odd. Thus, there is only one way in which both ab and cd are odd, which is setting a, b, c and d all to 1 (so p, q, r and s all to true.) For the former case, to make a product of two integers even, it suffices for at least one to be even, which can be done in three ways (both even OR a even and b odd OR a odd and b even). Thus, there are a total of nine ways (3 x 3) to make both ab and cd even. It follows that there are a total of 9 + 1 = 10 truth settings of p, q, r and s which make the expression above an even number. To be explicit, those 10 settings are:

p	q	r	s
F	F	F	F
F	F	F	T
F	F	T	F
F	T	F	F
F	T	F	T
F	T	T	F
T	F	F	F
T	F	F	T
T	F	T	F
T	T	T	T