<u>COT 3100 Final Exam - Part A (Relations, Functions) - 25 pts (4/24/2025)</u>
<u>Sections ≡ 1 (mod 200)</u>

<u>Please CLEARLY PRINT YOUR NAME IN CAPITAL LETTERS</u>

Last Name: ___________________ , First Name: _____________________

1) (10 pts) Let $R = \{(a, b) | \exists c \in Z^+ | ab = c^3\}$ be a relation defined over $Z^+ \times Z^+$. With proof determine if the relation is (a) reflexive, (b) irreflexive, (c) symmetric, (d) anti-symmetric and (e) transitive. Please clearly circle your choice and give a justification for your choice.

(a) Reflexive: Yes No

(b) Irreflexive: Yes No

(c) Symmetric: Yes No

(d) Anti-symmetric: Yes No

(d) Transitive: Yes No

2) (10 pts) Let $f(x) = ax^3 + bx^2 + cx + d$, for constants a, b, c and d. If $f(-1)$, $f(0)$ and $f(1)$ form an arithmetic sequence, with proof, determine the value of b. (Put a circle around your answer.)

3) (4 pts) How many injective functions exist $f: A \rightarrow B$, where $|A| = 4$ and $|B| = 8$. **Express your answer in prime factorized form.**

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4) (1 pt) Who composed Beethoven's 5^{th} Symphony? ________________________

COT 3100 Final Exam - Part B - 100 pts (4/24/2025)
Sections ≡ 1 (mod 200)

Please CLEARLY PRINT YOUR NAME IN CAPITAL LETTERS

Last Name: ___________________ , First Name: _____________________

1) (10 pts) The statement $\forall x \exists y[3x^2 + 2xy - 8y^2 = 0]$ over the domain of real numbers x and y is true. For most values of x, there exist 2 distinct values of y which make the statement true. **Determine what those values of y are, in terms of x.** For what value(s) of x does there only exist one value of y for which the statement is true?

2) (9 pts) Let A, B, C and D be finite sets of integers such that $(A \cup B) \subseteq (C \cap D)$. Each of the following claims are false. For each one, state a counter-example that proves the claim isn't true in all cases. (For once, no need for words. Just specify the counter-example fully in each of the cases and credit will be awarded based on whether you consistently filled in each set and whether or not what you have provided is a counter-example to the claim.)

(a) $A \subseteq D - B$

A = { } B = { }

C = { } D = { }

$A \cup B$ = { } $C \cap D$ = { }

$D - B$ = { }

(b) $(A \cap C) \subseteq (B \cap D)$

A = { } B = { }

C = { } D = { }

$A \cup B$ = { } $C \cap D$ = { }

$A \cap C$ = { } $B \cap D$ = { }

(c) $A - B \subseteq C - D$

A = { } B = { }

C = { } D = { }

$A - B$ = { } $C - D$ = { }

3) (10 pts) Find $65^{-1} \bmod 219$.

4) (12 pts) **Prove via induction on n**, that for all non-negative integers n,

$$\begin{pmatrix} 3 & 2 \\ 1 & 2 \end{pmatrix}^n = \begin{pmatrix} \frac{2}{3} \times 4^n + \frac{1}{3} & \frac{2}{3} \times 4^n - \frac{2}{3} \\ \frac{1}{3} \times 4^n - \frac{1}{3} & \frac{1}{3} \times 4^n + \frac{2}{3} \end{pmatrix}$$

5) (8 pts) Determine the **product** of the divisors of 1500. Since this number is very large, please express the number in **prime-factorized form.**

6) (10 pts) In COT 3100H this semester, 19 students out of 23 showed up to the final exam review (last day of class). All four of the students who didn't show up were in the bottom half of the grade distribution of the course (bottom 11 students). If **showing up to the final exam review and class grades were independent,** what would the probability of this occurring be?

(a) Please express your answer as a **fraction in lowest terms.**
(b) Please use your calculator to express this fraction as a decimal rounded to the nearest one thousandth.
(c) Draw a conclusion about whether or not these two items are likely to be independent or not.

(a) ______________ (b) _______________

Put your reasoning for (c) above.

7) (10 pts) When data is transmitted electronically, we can model the amount of time it takes as a discrete random variable as follows:

Number of Time Steps for Transmission	Probability
1	1/10
2	3/10
3	1/3
4	1/5

These probabilities don't add up to 1. In particular, if the transmission isn't complete after 4 time steps, the original transmission is terminated and a new copy is resent at that time. (Of course, it's possible this second transmission also fails, in which case a third one would be made 4 time steps later.)

Using this information, determine the expected number of time steps a transmission should take to get to its destination. Please express your answer as a fraction in lowest terms.

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8) (10 pts) A class has homework, quizzes and exams. Homework comprises 20% of the course grade, quizzes comprise 30% of the course grade and exams comprise 50% of the course grade. There are 5 homework assignments, each out of 50 points. There are 3 quizzes, each out of 25 points and there are 2 exams, each out of 100 points.

Currently, Elizah has earned 48 points and 47 points, respectively on the first two homework assignments, 20 points on the first quiz and 68 points on the first exam.

(a) What is Elizah's **actual** current grade in the class as a percentage? (Show your work and express your answer rounded to the nearest thousandth of a percentage point.)

(b) What would Webcourses report is Elizah's current grade in the class as a percentage? (Show your work and express your answer rounded to the nearest thousandth of a percentage point.)

(a) __________________________ (b) __________________________

9) (10 pts) Charlie is walking on the Cartesian plane, starting at (0, 0). He always walks a unit in the direction of the positive x-axis or a unit in the direction of the positive y-axis. His end destination is (13, 17). (Please express your answer using combination notation and anything else that is necessary.)

(a) (2 pts) In how many different ways can he complete his walk?

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(b) (4 pts) At (10, 4), there is a donut shop. Since Charlie is on a diet, he would like to avoid walking by the donut shop, because he knows he'll get tempted to break his diet if he can smell the delicious donuts. In how many ways can he make his walk while avoiding visiting (10, 4)?

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(c) The whole street with $x = 0$, from $y = 1$ through $y = 17$ is lined with tourists, which Charlie would like to avoid. In addition, the whole street with $x = 13$ from $y = 0$ through $y = 16$ is lined with even more tourists, which Charlie would like to avoid. In how many ways can he make his walk in this case? (For this part, there's no restriction on visiting (10, 4).)

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10) (10 pts) Akira made 5 shots out of 12 in a basketball game she played. If her probability of making each shot is independent of her probability of making the other shots, what is the probability that she never made two shots in a row? Please express you final answer as a fraction in lowest terms.

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11) (1 pt) The 2025 Boston Marathon was held on April 21, 2025. In which city did the marathon occur?

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Scratch Page – Please clearly mark any work you would like graded on this page.