COT 3100 Final Exam - Part B - 125 pts (7/30/2026)

Please CLEARLY PRINT YOUR NAME IN CAPITAL LETTERS

Last Name: ___________________ , First Name: _____________________

Logic (20 pts)

1) (8 pts) Complete the following truth table. Please use T for true and F for false and make each letter clear.

p	q	r	$p \rightarrow q$	$\bar{q} \rightarrow r$	$(p \rightarrow q) \wedge (\bar{q} \rightarrow r)$
F	F	F			
F	F	T			
F	T	F			
F	T	T			
T	F	F			
T	F	T			
T	T	F			
T	T	T			

2) (12 pts) Prove or disprove the following assertions using the universe of real numbers for both x and y. First clearly state if the assertion is true or false, followed by a justification for your answer.

(a) (6 pts) $\forall x \exists y [x + y = 0]$

(b) (6 pts) $\exists x \forall y [x + y = 0]$

Sets (20 pts)

3) (10 pts) Let A = {1, 4, 7, 9}, B = {2, 3, 4, 6, 9, 11} and C = {3, 5}. List the members of each of the following sets:

$A \cup B = \{$ $\}$

$A - B = \{$ $\}$

$C \times (B - A) = \{$ $\}$

$\wp(B - A - C) = \{$ $\}$

4) (10 pts) Prove the following assertion for all sets A, B and C:

$$\text{if } B \cap C \subseteq A \text{ and } A \subseteq B \cup C, \text{ then } \big((B \cup C) - A\big) \cap (B \cap C) = \emptyset.$$

Number Theory (20 pts)

5) (5 pts) Provide the quotient and remainder for each of the problems when dividing a by b

i) a = 12, b = 7 q = ____ , r = ____

ii) a = 114, b = 202 q = ____ , r = ____

iii) a = 10543, b = 1000 q = ____ , r = ____

iv) a = 97, b = 8 q = ____ , r = ____

v) a = 142, b = 13 q = ____ , r = ____

6) (15 pts) Let a be a positive integer and p be a prime number such that gcd(a, p) = 1. Consider the following set, S, of p-1 integers: S = {a, 2a, 3a, 4a, ..., (p-1)a}.

Prove the following assertions. In your work, you may assume the following lemma: For all primes p and positive integers a and b, if p | (ab) then either p | a or p | b.

(a) (8 pts) That each integer in the set leaves a different remainder when divided by p. (Hint: Use proof by contradiction.)

(b) (5 pts) That no integer in the set is divisible by p.

(c) (2 pts) Based on these findings, draw a conclusion about the set created by replacing each item in S with its remainder when divided by p.

Induction (20 pts)

7) (10 pts) This is a two part question where you'll prove a summation formula.

(a) (2 pts) Plug in small values of n to determine a guess for the closed-form solution, in terms of n, for the following sum: $\sum_{i=1}^{n} \frac{1}{i(i+1)}$, for all positive integers n.

(b) (8 pts) Use Mathematical Induction to prove your guess from part (a). (Note: You can only get credit for this part if your guess from part (a) is correct.)

8) (10 pts) Use mathematical induction on n to prove for all positive integers n that

$$\begin{bmatrix} 2 & 0 \\ 1 & 1 \end{bmatrix}^n = \begin{bmatrix} 2^n & 0 \\ 2^n - 1 & 1 \end{bmatrix}$$

Probability (20 pts)

9) (10 pts) Of all stocks available, Warren Buffet invests in 25% of them. Given that he's invested in a company, there is a 70% chance it will make a profit. However, if he hasn't invested in a company, there is only a 45% chance it will make a profit.

(a) (6 pts) What is the probability a randomly chosen stock out of all stocks available makes a profit? (Please answer as a fraction in lowest terms.)

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(b) (4 pts) Given that a company has made a profit, what is the probability that Warren Buffet invested in it? (Again, leave your answer as a fraction in lowest terms.)

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10) (10 pts) Jim lives at the corner of 1^{st} Street and 1^{st} Avenue, which is at the southeast corner of town. If he walks one block to the north from where he lives, he'll end up at the intersection of 2^{nd} Street and 1^{st} Avenue, and if he walks one block to the west from where he lives, he'll arrive at the intersection of 1^{st} Street and 2^{nd} Avenue. Jim needs to walk to his office, which is at the corner of 13^{th} Street and 9^{th} Avenue. He plans on walking individual blocks, always moving either north or west. He picks one of these possible paths randomly. What is the probability that he visits the intersection of 2^{nd} Street and 2^{nd} Avenue during his journey? **(Note: For 7 pts leave your answer in powers, factorials, combinations. To receive the full 10 points, express your answer as a fraction in lowest terms.)**

Relations and Functions (20 pts)

11) (12 pts) It was mentioned in class that when given relations $R \subseteq A \times B$, $S \subseteq B \times C$ and $T \subseteq B \times C$, that is not necessarily true that $(S \circ R) \cap (T \circ R) \subseteq (S \cap T) \circ R$.

(a) (8 pts) Start attempting a direct proof of this assertion and clearly explain where you get stuck and why you can't proceed to complete the proof.

(b) (4 pts) Utilizing the intuition gained from your work in (a), or otherwise, devise a counter-example, clearly stating sets A, B, C and relations R, S and T that disproves the assertion $(S \circ R) \cap (T \circ R) \subseteq (S \cap T) \circ R$. (Namely, show that there is at least one example of relations R, S and T for which this claim is NOT true.)

12) (8 pts) Let $f(x) = 2x^2 - 16x + 4$, with a domain of all real x, x ≤ 4.

(a) (6 pts) Find $f^{-1}(x)$. Put a box around your answer.

(b) (2 pts) What are the domain and range of $f^{-1}(x)$?

Freebie (5 pts)

13) (5 pts) In the video game "Oregon Trail", players start in Independence, Missouri and attempt to trek to what state via wagon? (According to Wikipedia's list of famous video games, this one was released first, all the way back in 1971, even before I was born!)

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Scratch Page – Please clearly mark any work you would like graded on this page.