Summer 2026 COT 3100 Final Exam - Part A (Counting) - 25 pts (7/30/2026) Solutions

1) (10 pts) If a, b, c, d, e and f are all required to be non-negative integers, how many solutions does the equation

$$a + 2b + 2c + 2d + 2e + 2f = 3100$$

have? Please leave your answer in terms of powers, factorials and combinations.

Solution

If we solve for a we see:

$a = 2(1550 - b - c - d - e - f)$

Since all of the variables are integers, it follows that $2 \mid a$. Let $a = 2a'$ where a' is a non-negative integer. Now we are looking for integer solutions to the equation:

$2a' + 2b + 2c + 2d + 2e + 2f = 3100$
$a' + b + c + d + e + f = 1550$

This is an instance of combinations with repetition with n = 1550 and r = 6. It follows that the number of possible non-negative solutions to this equation is $\binom{1550+6-1}{6-1} = \binom{\mathbf{1555}}{\mathbf{5}}$**.**

Grading: 1 pt for recognizing that a must be even,
2 pts for the justification why a must be even,
1 pt substitute a = 2a' (or whatever letter)
2 pts to get to a'+b+...+f = 1550
2 pts for stating this is just combinations with repetition
2 pts for the final answer

Note: Partial Credit for these answers
1550! – 1 pt
1550^5, C(3100,5), C(3100, 6) – 2 pts for each of these
C(3106, 6) – 3 pts
C(1550,5)x1550 – 5 pts
C(1554, 4) – 7 pts

2) (14 pts) Each of the following parts will deal with permutations of the letters in the word "TEMPERAMENT". Please leave your answers in terms of powers, factorials, and combinations.

(a) (2 pts) How many total permutations are there of the letters in the word "TEMPERAMENT"?

Here are the letter counts: 2 Ts, 3 Es, 2 Ms, 1 P, 1 R, 1 A, and 1 N. Applying the permutation formula, we get the result:

$\frac{\mathbf{11!}}{\mathbf{2!3!2!}}$ **, Grading: no explanation required, 1 pt numerator, 1 pt denominator.**

(b) (4 pts) How many of the permutations in part (a) do NOT have consecutive vowels?

Use the consonants as separators: __ T __ T __ M __ M __ P __ R __ N __. There are 8 slots to place vowels and we must choose 4 of those slots to place the vowels, which we can do in $\binom{8}{4}$ ways. Once the vowels are placed, they can be permuted in $\frac{4!}{3!}$ ways. (This is just 4.) The consonants can be permuted in $\frac{7!}{2!2!}$ ways. It follows that the total number of permutations without consecutive vowels is

$\binom{\mathbf{8}}{\mathbf{4}} \times \frac{4!}{3!} \times \frac{7!}{2!2!} = \binom{\mathbf{8}}{\mathbf{4}} \times \mathbf{7!}$ **, Grading: 1 pt for** $\binom{\mathbf{8}}{\mathbf{4}}$**, 1 pt for** $\frac{4!}{3!}$**, 1 pt for** $\frac{7!}{2!2!}$**, 1 pt multiply (Note: Different approaches may earn partial on a case by case basis.)**

(c) (8 pts) How many permutations of "TEMPERAMENT" contain the substring "ME"?

Our usual technique is to create a superletter "ME", so we have 10 "tiles", of which 2 are T and 2 are E. There are a total of $\frac{10!}{2!2!}$ of these permutations.

The problem with this count is that there are two Ms and two Es, so there are some of the permutations above that we counted that have ME twice. This means that each of those permutations were counted twice, once for the first ME and a second time for the second ME. Thus, we must just subtract out all of the permutations with two copies of ME. So, just make two super letters "ME", leaving 9 "tiles" total, of which there are 2 Ts. There are a total of $\frac{9!}{2!}$ of these permutations.

It follows that the final answer is $\frac{\mathbf{10!}}{\mathbf{2!2!}} - \frac{\mathbf{9!}}{\mathbf{2!}} = \frac{9!}{2!}\left(\frac{10}{2} - 1\right) = \frac{9!}{2!} \times 4 = \mathbf{2(9!)}$

Grading: 2 pts for $\frac{10!}{2!2!}$**, 2 pts for recognizing that this isn't correct due to over-counting, 4 pts for the fix. Students don't need to do any of the simplification I did, they can just answer** $\frac{\mathbf{10!}}{\mathbf{2!2!}} - \frac{\mathbf{9!}}{\mathbf{2!}}$**.**

3) (1 pt) What color is the logo for Blue Bottle Coffee? **BLUE (Give to All!)**