Summer 2026 COT 3100 Section 1 Exam #2 – Part 1 (Number Theory) – 45 pts (6/30/2026)

Last Name: ______________________ **First Name:** _____________________

1) (10 pts) Prove or disprove: Let a, b, c and d be positive integers. If gcd(a, b) = 1 and gcd(c, d) = 1, then gcd(ac, bd) = 1.

2) (5 pts) With proof, find all of the integer solutions, (x, y), to the equation 91x + 52y = 142.

3) (15 pts) With proof, find all integer solutions, (x, y), to the equation $765x + 312y = 12$.

4) (10 pts) Unlike some previous offerings of COT 3100, this time I've shown the class some code. While I won't ask students to write code on an exam, if students have been carefully paying attention in class, they should be able to understand what a function is doing if it's performing a task based on what is taught in class. In English (do NOT try to translate the code literally), explain as clearly and succinctly as possible what the following function in Python does. (Note: two forward slashes is integer division, and 3 arguments in a for loop signifies the starting point, ending point exclusive, and how much to increment by.)

```
def whatdoesitdo(n):

    a = [True]*(n+1)
    for i in range(2, n+1):

        if a[i]:
            for j in range(2*i, n+1, i):
                a[j] = False

            x = 0
            y = n
            while y > 0:
                x += y//i
                y = y//i
            print("(",i,"^",x,") *", sep="", end=" ")

    print()
```

Write Answer Below

5) (5 pts) What is the product of the divisors of 144? Please express your answer in prime factorized form. (Note: You weren't taught how to do this specific task, but if you use the information you were taught, you can derive the appropriate formula for this task. **Full credit will only be given if this task is done using information taught rather quickly and not using brute force**.)

Summary 2026 COT 3100 Section 1 Exam #2 – Part 2 (Induction) – 55 pts (6/30/2026)

Last Name: ______________________ First Name: _____________________

6) (10 pts) Prove, using induction on n, for all non-negative integers, n that

$$\sum_{i=0}^{n}(i(i+1)) = \frac{n(n+1)(n+2)}{3}$$

7) (15 pts) Prove, using induction on n, for all positive integers, n that

$$\sum_{i=1}^{2^n-1} (log_2 i) \geq n(2^n) - 2^{n+1} + 2$$

8) (13 pts) Prove via induction on n, for all non-negative integers n, that $13 \mid (5^{2n} + 12^{n+1})$.

9) (12 pts) Let F_n denote the n^th^ Fibonacci number. Prove via strong induction on n that the number of strings of a's and b's of length n that do NOT have 2 consecutive b's equals F_{n+2}, for all positive integers n. (Note: $F_1 = 1$, $F_2 = 1$, $F_n = F_{n-1} + F_{n-2}$, for all $n \geq 3$.) Please use two base cases. (Note: for example, for n = 3, there are five such strings: aaa, aab, aba, baa and bab. Notice that abb, bba and bbb each have at least one occurrence of 2 consecutive b's, which is why they weren't listed.) This is a constructive proof, fairly similar in its nature to both the tromino and square partition problems shown in class.

10) (5 pts) Polar Seltzer Sparkling Water uses what animal on its logo?

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Scratch Page – Please clearly mark any work on this page you would like graded.