Summer 2026 COT 3100 Section 1 Exam #1 - Part 1 (Logic) - 50 pts (6/2/2026)

Solutions

1) (8 pts) Complete the following truth table. Fill in each empty cell with either the letter F (for false) or T (for true).

p	q	r	$p \to \bar{q}$	$r \lor q$	$(p \to \bar{q}) \land (r \lor q)$
F	F	F	T	F	F
F	F	T	T	T	T
F	T	F	T	T	T
F	T	T	T	T	T
T	F	F	T	F	F
T	F	T	T	T	T
T	T	F	F	T	F
T	T	T	F	T	F

Grading: 1 pt per row, row has to be completely correct to get the point.

2) (4 pts) How many combinations of truth settings of the variables p, q, r, s and t make the expression $(p \lor q) \land (r \lor s \lor t)$ true? (Note: There are a total of 32 possible truth settings of the five variables, so the answer must be in between 0 and 32, inclusive.)

There are three truth settings that make $p \lor q$ True (anything but setting both to false) and there are seven truth settings that make $r \lor s \lor t$ True (again, anything but setting all three to false). We can combine any selection of 3 truth settings for p/q with any of the 7 truth settings of r/s/t, so we need to multiply the two numbers to get 21 possible truth settings that make the given expression true.

21 **Grading: 1 pts for 3 settings p/q, 2 pts 7 settings s/t/u, 1 pts to multiply to correct result.**

3) (8 pts) Prove or disprove the following assertion. The chosen universe of numbers for both x and y (they are different) are described in the problem below:

$$\forall x \in R, x > 1 \exists y \in R, y > 0 [\frac{1}{x} + \frac{1}{y} = 4x]$$

This is true. Let's take the given expression and solve for y with the assumption that y > 0 and x > 1:

$$\frac{1}{x} + \frac{1}{y} = 4x \;\to\; \frac{1}{y} = 4x - \frac{1}{x} \;\to\; y = \frac{1}{4x - \frac{1}{x}} \;\to\; y = \frac{1}{\frac{4x^2-1}{x}} \to y = \frac{x}{(2x-1)(2x+1)}$$

Thus, we've shown that for any positive real number x greater than 1, the positive value of y that exists to make the given statement true is $\frac{x}{(2x-1)(2x+1)}$. Notice that for any x > 1, all three terms in the fraction are positive, so the corresponding value of y is positive, also inferring that there was no division by zero in any of our work.

Grading: 2 pts stating true, 4 pts for algebra solving for y (give partial as you see fit), 2 pt for explaining why no illegal mathematical operations were done for universes given.

4) (18 pts) Using the laws of logic show that the two following logical expressions are equivalent:

(1) $(q \to p) \wedge [(r \vee (\bar{p} \wedge r)) \vee ((\bar{r} \vee \bar{q}) \wedge (\bar{r} \vee q))]$ (2) $p \vee \bar{q}$

For the purposes of grading, please do NOT skip steps such Associative and Commutative. Note: More rows are given than necessary.

Step	Reason
$(q \to p) \wedge [(r \vee (\bar{p} \wedge r)) \vee ((\bar{r} \vee \bar{q}) \wedge (\bar{r} \vee q))]$	Given
$(q \to p) \wedge [(r \vee (\bar{p} \wedge r)) \vee (\bar{r} \vee (\bar{q} \wedge q))]$	Distributive Law
$(q \to p) \wedge [(r \vee (\bar{p} \wedge r)) \vee (\bar{r} \vee (q \wedge \bar{q}))]$	Commutative Law
$(q \to p) \wedge [(r \vee (\bar{p} \wedge r)) \vee (\bar{r} \vee (F))]$	Inverse Law
$(q \to p) \wedge [(r \vee (\bar{p} \wedge r)) \vee \bar{r}]$	Identity Law
$(q \to p) \wedge [(r \vee \bar{r}) \vee (\bar{p} \wedge r)]$	Associative Law
$(q \to p) \wedge [T \vee (\bar{p} \wedge r)]$	Inverse Law
$(q \to p) \wedge [(\bar{p} \wedge r) \vee T]$	Commutative Law
$(q \to p) \wedge T$	Domination Law
$(q \to p)$	Identity Law
$\bar{q} \vee p$	Implication Identity
$p \vee \bar{q}$	Commutative Law

Grading: 2 pts per step, allow up to 2 Commutative steps skipped without taking off any credit. Grade additive for incomplete responses and subtractive for those that are complete but have errors. The 2 pts per step should be broken into 1 pt for the step, 1 pt for the reason.

5) (12 pts) Prove the following argument via the Rules of Inference.

$$p \to q$$
$$r \to s$$
$$p \vee r \vee t$$
$$q \to u$$
$$\bar{t}$$
$$s \to u$$

$$\therefore u$$

Note: More rows are given than necessary. Make sure to list each given step explicitly as they are worth points.

Number	Step	Reason
1	$p \vee r \vee t$	Given
2	$\bar{t}$	Given
3	$p \vee r$	Disjunctive Syllogism with #1 and #2
4	$p \to q$	Given
5	$r \to s$	Given
6	$q \vee s$	Constructive Dilemma with #4, #5 and #3
7	$q \to u$	Given
8	$s \to u$	Given
9	$(q \vee s) \to u$	Proof by Cases with #7 and #8
10	u	Modus Ponens with #6 and #9

Grading: 1 pt for each given step (pretty sure all are needed)
1 pt for each Step, 2 pts total for each of the reasons

6) (5 pts) If $A = \{2, 7, 8\}$ and $B = \{4, 5\}$, the set $\wp(A \times B)$ has 64 elements. List any five elements from this set.

There are an insane number of correct answers here. Each element of $\wp(A \times B)$ is a set of ordered pairs. Of course, the empty set is an element of every power set, so that's the shortest one to write out. In addition, there are six more sets with just 1 element that are in this power set, so if you are lazy, you could write:

$\emptyset$, {(2,4)} , {(2,5)} , {(7,4)} , {(7,5)}

Grading: 1 pt per set listed, type of object must be correct to earn the point. (Only clearly listed sets of ordered pairs can get credit.)

7) (10 pts) Use Set Laws to show that the two sets below are equivalent.

(1) $[(C \cap D) \cup (C \cap A \cap D)] \cap [(B \cup C) \cap (B \cup D)]$ (2) $C \cap D$

Step	Reason
$[(C \cap D) \cup (C \cap A \cap D)] \cap [(B \cup C) \cap (B \cup D)]$	Given
$[(C \cap D) \cup (C \cap A \cap D)] \cap [B \cup (C \cap D)]$	Distributive Law
$[(C \cap D) \cup (C \cap D \cap A)] \cap [B \cup (C \cap D)]$	Commutative Law
$(C \cap D) \cap [(B \cup (C \cap D)]$	Absorption Law
$(C \cap D) \cap [(C \cap D) \cup B]$	Commutative Law
$C \cap D$	Absorption Law

Grading: 2 pts per step (1 pt step, 1 pt reason) for upto 5 lines leading to the solution (lines that are valid but don't help to get to the solution do not count), and correct response gets full credit, if something works fairly differently and is mostly correct, use subtractive grading but make sure the final number of points is reasonable.

8) (10 pts) Prove or Disprove: For all sets A and B, $\wp(A \cup B) = \wp(A) \cup \wp(B)$.

This claim is false. Consider the following counter-example:

$A = \{1\}$, $B = \{2\}$

In this case, $A \cup B = \{1,2\}$, and by definition of power set, $\{1,2\} \in \wp(A \cup B)$. But, notice that $\{1,2\} \notin \wp(A)$, since $2 \notin A$ and $\{1,2\} \notin \wp(B)$, since $1 \notin B$. It follows that $\{1,2\} \notin \wp(A) \cup \wp(B)$, meaning that for this example , $\wp(A \cup B) \neq \wp(A) \cup \wp(B)$.

Grading: Max 2 pts for proof attempt
4 pts for stating it's false
4 pts for a counter-example that works (0 pts if it doesn't)
2 pts for explaining why it's a counter-example

Note: An analysis of the number of elements in the set on the LHS and the maximum number of elements in the set on the RHS can work as well, but this is somewhat tricky to handle without a specific counter-example.

9) (10 pts) Prove or Disprove: For all sets A, B and C: if $A - B = A - C$, then $A \cap B \cap C = B \cap C$.

This is false. Consider the following counter-example:

$A = \emptyset$, $B = \{1\}$, $C = \{1\}$.

In this case, $A - B = A - C = \emptyset$, so the if clause is true. But, $A \cap B \cap C = \emptyset$ while $B \cap C = \{1\}$, so these two sets aren't equal in this case so the then clause is false.

Grading: Max 2 pts for proof attempt
4 pts for stating it's false
4 pts for a counter-example that works (0 pts if it doesn't)
2 pts for explaining why it's a counter-example

10) (10 pts) Let A, B and C be sets such that $|A \cup B \cup C| = 35$, $|A \cup B| = 26$, $|A \cap B| = 0$, and $|B \cup C| = 32$. Determine the value of $|A \cap \bar{B} \cap \bar{C}|$.

Let's write down the Inclusion-Exclusion Principle for 3 sets and see what information we can find. Before we do, note that since $|A \cap B| = 0$, it must be the case that $|A \cap B \cap C| = 0$, since if there are no common elements between A and B, there can be no elements that belong to all three of A, B and C.

$$|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |A \cap C| - |B \cap C| + |A \cap B \cap C| \textbf{ (2 pts)}$$
$$35 = |A| + |B| + |C| - 0 - |A \cap C| - |B \cap C| + 0 \textbf{ (2 pts)}$$

Now, recall that via the Inclusion-Exclusion Principle for 3 sets, $|B \cup C| = |B| + |C| - |B \cap C|$, and make this substitution in the equation above:

$$35 = |A| + (|B| + |C| - |B \cap C|) - |A \cap C|$$
$$35 = |A| + |B \cup C| - |A \cap C|$$
$$35 = |A| + 32 - |A \cap C|$$
$$|A| - |A \cap C| = 3 \textbf{ (3 pts)}$$

Now, notice that we seek $|A \cap \bar{B} \cap \bar{C}|$. First note that since $|A \cap B| = 0$, that means that A and B share no items, which means that all items in A belong to $\bar{B}$. It follows that both $A \subseteq \bar{B}$ and that $A \cap \bar{B} = A$. This means that $|A \cap \bar{B} \cap \bar{C}| = |A \cap \bar{C}|$.

Finally note that $A = A \cap (C \cup \bar{C}) = (A \cap C) \cup (A \cap \bar{C})$, with $(A \cap C)$ and $(A \cap \bar{C})$ being disjoint. This means that $|A| = |A \cap C| + |A \cap \bar{C}|$. Now, we can substitute for $|A|$ in the equation above to yield:

$$|A| - |A \cap C| = 3$$
$$|A \cap C| + |A \cap \bar{C}| - |A \cap C| = 3$$
$$|A \cap \bar{C}| = 3$$

It follows that $|A \cap \bar{B} \cap \bar{C}| = 3$ as well. **(Grading: Last 3 points to argue that $|\boldsymbol{A}| - |\boldsymbol{A} \cap \boldsymbol{C}| = |\boldsymbol{A} \cap \bar{\boldsymbol{B}} \cap \bar{\boldsymbol{C}}|$ for this particular case. Doesn't need to be quite as rigorous as what's above.)**

11) (4 pts) What do I ask students in the class to do at the beginning of each lecture (so far)?

<u>Make sure all students are sitting in the front half of the lecture hall.</u> **Grading: Up to my discretion!**

12) (1 pt) Who wrote the autobiography, <u>The Narrative of the Life of Frederick Douglass</u>?

<u>Frederick Douglass</u> **(Grading: Give to All)**