

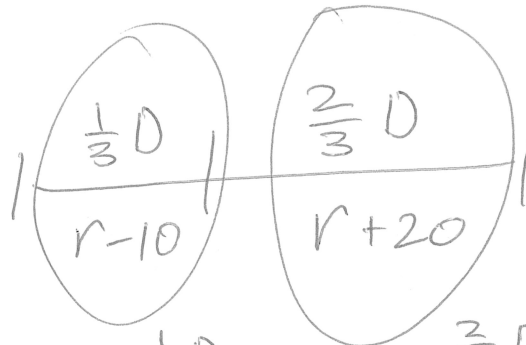
Part A

1) Make Drawing

$D = \text{whole } D$

$D = rt$

$t = \frac{D}{r}$



$t_1 = \frac{\frac{1}{3}D}{r-10}, t_2 = \frac{\frac{2}{3}D}{r+20}$

$\frac{D}{r} = \frac{\frac{1}{3}D}{r-10} + \frac{\frac{2}{3}D}{r+20}$

$\frac{1}{r} = \frac{1}{3(r-10)} + \frac{2}{3(r+20)}$

Key ideas
 Create D
 find 2
 expressions
 for time
 of trip

2) Key assumption const ratio btw wrong clock + the actual time. [Jessie's 10 min = 9 min real time]

real = 9:36 pm - 2 pm = 480 - 24 min

Jessie = 10 pm - 2 pm = 480 min

Ratio: $\frac{480}{456} = \frac{20}{19}$

19 real days = 20 days Jessie's Clock

3) Testing base ~~case~~ change

4) Tempting to set up sys eqn a, d .

BUT \Rightarrow this is going computationally expensive

$$\begin{array}{r}
 \begin{array}{l}
 \xrightarrow{d} \\
 a_{26} + a_{27} + a_{28} + \dots \\
 \xrightarrow{d} \\
 - a_1 + a_2 + a_3 + \dots
 \end{array}
 \end{array}
 \qquad
 \begin{array}{l}
 a_{50} = 3138 \\
 a_{25} = 638
 \end{array}$$

$$25d + 25d + 25d + \dots$$

$$25d_1 = 2500$$

$$625d = 2500$$

$$d = 4$$

5) How many times does 8 divide into $100!$

formula \Rightarrow only works for primes...

answer instead: how many times does 2 divide into $100!$

$$\begin{array}{r}
 2 \overline{)100} \\
 \underline{2 \overline{)50}} \\
 2 \overline{)25} \\
 \underline{2 \overline{)12}} \\
 2 \overline{)6} \\
 \underline{2 \overline{)3}} \\
 2 \overline{)1} \\
 \underline{\phantom{2 \overline{)1}}} \\
 1
 \end{array}$$

$$50 + 25 + 12 + 6 + 3 + 1 = 97$$

$$100! = 2^{97} \times \dots$$

$$8 = 2^3$$

$$\left\lfloor \frac{97}{3} \right\rfloor = 32$$

Part B

1) Look to see "where" simplifications can be made. Look for same var appearing multiple times.

2) Testing similar lecture question

Key: separator trick

14 separators

15 gaps

$_ M _ P_i _ M _ P_i _ \dots _$

$\binom{15}{10} \rightarrow$ choose location palms

$\binom{14}{6} \rightarrow$ choose location maples

* I like asking questions that end up being combinations!

3) Key ideas: De Morgan's Law in negation

if $A \not\subseteq E \Rightarrow \exists x \in A \wedge x \notin E$.

if $x \in A \wedge x \notin E$ and $y \in B$

then $(x, y) \in A \times B \wedge (x, y) \notin E \times F$

Only way to make part B work is
 $B = \emptyset$

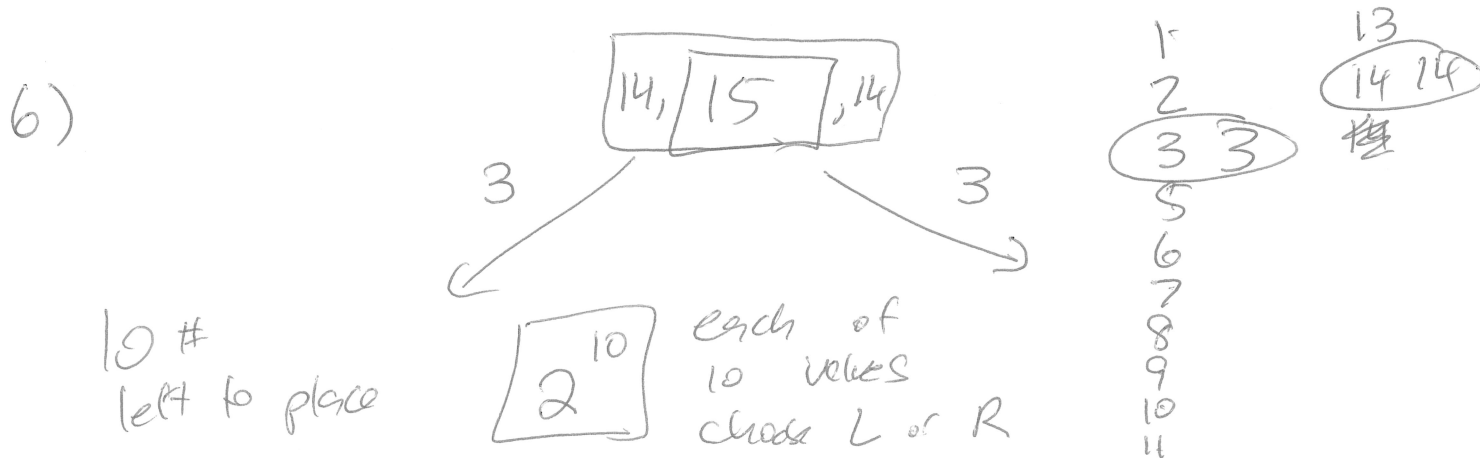
Similarly only way to make part C work is $A = \emptyset$

4) B) $A = \{1\}$ $B = C = D = E = F = \emptyset$
 $A \notin E$ but $A \times B \cup C \times D = \emptyset$
 $E \times F = \emptyset$

C) $B = \{1\}$, $A = C = D = E = F = \emptyset$

4) Linear Eqn Solver
 MOD INVERSE, etc.

5) $162,000 = 162 \times 1000$
 $= 2 \times 81 \times 2^3 \times 5^3$
 $= 2^4 \times 3^4 \times 5^3$
 Sum Div $= \frac{2^5 - 1}{2 - 1} \times \frac{3^5 - 1}{3 - 1} \times \frac{5^4 - 1}{5 - 1}$
 $= 31 \times \frac{242}{2} \times \frac{624}{4}$
 $= 31 \times 121 \times 156$
 $= 31 \times 11^2 \times 4 \times 39$
 $= 31 \times 11^2 \times 2^2 \times 3 \times 13$



b.c. $n=0$ prob of tossing even # of Heads w/o tosses is 1.

$$RHS = \frac{1 + \left(\frac{1}{3}\right)^0}{2} = \frac{1+1}{2} = 1 \quad \checkmark$$

base case holds

i.h. Assume for an arbitrarily chosen non-neg int $n \geq k$ that the probability of getting same parity as k # of heads

is $\frac{1 + \left(\frac{1}{3}\right)^k}{2}$.

i.s Prove that the probability of getting the same parity as $k+1$ # of heads is $\frac{1 + \left(\frac{1}{3}\right)^{k+1}}{2}$.

$$P(\text{parity } k+1 \text{ \# head in } k+1 \text{ tosses}) \\ = P(\text{parity of } k \text{ heads in } k \text{ tosses}) \times P(H) + \\ = P(\text{parity of } k+1 \text{ heads in } k \text{ tosses}) \times P(T)$$

$$= \frac{1 + \left(\frac{1}{3}\right)^k}{2} \times \frac{2}{3} + \frac{1 - \left(\frac{1}{3}\right)^k}{2} \times \frac{1}{3}$$

$$= \frac{1}{3} + \frac{\left(\frac{1}{3}\right)^k}{3} + \frac{1}{6} - \frac{\left(\frac{1}{3}\right)^{k+1}}{2}$$

$$= \frac{1}{2} + \left(\frac{1}{3}\right)^{k+1} - \frac{\left(\frac{1}{3}\right)^{k+1}}{2}$$

$$= \frac{1 + \left(\frac{1}{3}\right)^{k+1}}{2}$$

$$8) b.c \quad n=1 \quad \text{LHS} = f'(x) = ax + b \quad \checkmark$$

$$\text{RHS} = a^1 x + \left(\frac{a^1 - 1}{a-1} \right) \times b$$

$$= ax + b \quad \checkmark$$

base case holds

i.h. Assume for an arbitrarily chosen positive integer $n=k$ that $f^k(n) = a^k x + \left(\frac{a^k - 1}{a-1} \right) b$

i.s Prove for $n=k+1$ that $f^{k+1}(n) = a^{k+1} x + \left(\frac{a^{k+1} - 1}{a-1} \right) b$

$$f^{k+1}(n) = f(f^k(n))$$

$$= f\left(a^k x + \left(\frac{a^k - 1}{a-1}\right) b\right) \text{ using IH}$$

$$= a \left[a^k x + \left(\frac{a^k - 1}{a-1}\right) b \right] + b$$

$$= a^{k+1} x + \frac{a(a^k - 1)}{a-1} \times b + \frac{b(a-1)}{a-1}$$

$$= a^{k+1} x + \frac{b[a^{k+1} - a + a - 1]}{a-1}$$

$$= a^{k+1} x + \left(\frac{a^{k+1} - 1}{a-1} \right) \times b$$